ENGINEERING MATHEMATICS -I

LECTURE NOTE

Based on New syllabus (2018-19). circulated by SCTE&VT, Odisha for 1st and 2nd Semester Diploma Engineering courses approved by AICTE,New Delhi

CONTENT

Unit	Topic
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Determinant And Matrix

Dt→06th July 2017 (Thursday)

* Determinant - To find the solution of simultaneously equation.

e.g → $a_1x + b_1y = 0$ ——— (I)

$a_2x + b_2y = 0$ ——— (II)

from eqn (I)

$a_1x = -b_1y$

$\Rightarrow x/y = \frac{-b_1}{a_1}$ ——— (III)

from eqn (II)

$a_2x + b_2y = 0$

$a_2x = -b_2y$

$x/y = \frac{-b_2}{a_2}$ ——— (IV)

from eqn (III) & (IV) we get

$\frac{-b_1}{a_1} = \frac{-b_2}{a_2}$

$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2}$

$\Rightarrow a_2b_1 = a_1b_2$

$\Rightarrow a_1b_2 - a_2b_1 = 0$ ——— (V)

Now eqn (I) can be written as

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0$$

Here a_1, b_1 are elements of 1st row (R_1)

a_2, b_2 are elements of 2nd row (R_2).

$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ are elements of 1st column (C_1).

$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ are elements of 2nd column (C_2).

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In determinants no. of rows is equal to no. of columns.

$a_2\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is the 2nd order determinant.

$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is the 3rd order determinant.

Similarly if we solve the three simultaneously

i.e $a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$a_3x + b_3y + c_3 = 0$

eliminating x & y from above three eq^n:

we get

$a_1(b_1c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - b_2a_3) = 0$

this can be written as in determinant form.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

This is the 3rd order determinant.

$$a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} +a_1 & -b_1 & +c_1 \\ -a_2 & +b_2 & -c_2 \\ +a_3 & -b_3 & +c_3 \end{vmatrix}$$

eg Find the value of determinant.

$$\begin{vmatrix} 2 & -3 & 5 \\ 7 & -8 & 4 \\ 9 & -10 & 5 \end{vmatrix}$$

$$= 2\begin{vmatrix} -8 & 4 \\ -10 & 5 \end{vmatrix} - (-3)\begin{vmatrix} 7 & 4 \\ 9 & 5 \end{vmatrix} + 5\begin{vmatrix} 7 & -8 \\ 9 & -10 \end{vmatrix}$$

$$= 2(-40 - 4(-10)) + 3(35 - 36) + 5(-70 - (-72))$$

$$= 2(-40 + 40) + 3(-1) + 5(-70 + 72)$$

$$= 2(0) - 3 + (5 \times 2)$$

$$= 0 - 3 + 10$$

$$= 7.$$

☆ Properties:-

(1) Property-1:-

If rows are changed to columns or columns are change to rows the value of determinant is unchanged.

eg $\begin{vmatrix} 2 & 5 \\ 7 & 3 \end{vmatrix} = 6 - 35 = -29$

$\begin{vmatrix} 2 & 7 \\ 5 & 3 \end{vmatrix} = 6 - 35 = -29$ (row change to column)

$\begin{vmatrix} 2 & 7 \\ 5 & 3 \end{vmatrix} = 6 - 35 = -29$ (column change to row).

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(2) **Property-2:-**

→ If any two adjacent rows or (columns) are interchanged, then absolute values are same and sign is reverse.

e.g→

$$\Delta = \begin{vmatrix} 2 & 5 \\ 6 & 8 \end{vmatrix} = 16 - 30 = -14$$

row change

$$\Delta_1 = \begin{vmatrix} 6 & 8 \\ 2 & 5 \end{vmatrix} = 30 - 16 = 14$$

Column change

$$\Delta_2 = \begin{vmatrix} 5 & 2 \\ 8 & 6 \end{vmatrix} = 30 - 16 = 14$$

$$\Delta = \begin{vmatrix} 2 & 5 & 6 \\ 8 & 9 & 10 \\ 3 & 2 & 1 \end{vmatrix} = 2(9-20) - 5(8-30) + 6(16-27)$$

$$= 2 \times (-11) - 5(-22) + 6(-11)$$

$$= -22 + 110 - 66$$

$$= -88 + 110$$

$$= 22$$

$$\Delta_1 = \begin{vmatrix} 2 & 5 & 6 \\ 3 & 2 & 1 \\ 8 & 9 & 10 \end{vmatrix}$$

R1 & R3 are interchanged

$$= 2(20-9) - 5(30-8) + 6(27-16)$$

$$= 2 \times 11 - 5 \times 22 + 6 \times 11$$

$$= 22 - 110 + 66$$

$$= 88 - 22$$

(3) **Property-3:-**

→ If any two rows or (columns) are identical. (corresponding elements are same), then the value of determinant is zero.

e.g

$$\begin{vmatrix} a & b & c \\ d & e & f \\ a & b & c \end{vmatrix} = 0$$

R_1 is identical to R_3.

e.g

$$\begin{vmatrix} 2 & 3 & 4 \\ 8 & 14 & 20 \\ 2 & 3 & 4 \end{vmatrix} = 2(56-60) - 3(32-40) + 4(24-24)$$

$$= 2 \times (-4) - 3(-8) + 4(-4)$$

$$= -8 + 24 - 16$$

$$= 0$$

or

$$\begin{vmatrix} a & a & d \\ b & b & e \\ c & c & f \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 2 & 8 \\ 3 & 3 & 14 \\ 4 & 4 & 20 \end{vmatrix}$$

$$= 2(60-56) - 2(60-56) + 8(12-12)$$

$$= 8 \times 4 - (2 \times 4) + 8(0)$$

$$= 8 - 8 + 0$$

$$= 0$$

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3. If any two rows or coloumn are proportional then the value of determinant is zero.

Eg $\begin{vmatrix} 2 & 3 & 4 \\ 8 & 14 & 20 \\ 4 & 6 & 8 \end{vmatrix} = 0$.

$2(112-120) - 3(64-80) + 4(48-56)$

$= (2 \times -8) - 3(-16) + 4(-8)$

$= -16 + 48 - 32$

$= 48 - 48$

$= 0$

4. Property-4:-

→ If each element of a rows (coloumn) of a determinant is multiplied by a constant 'k', then it's value get multiplied by 'k'.

Eg $\Delta = \begin{vmatrix} 2 & 4 & 5 \\ 3 & 8 & 9 \\ -6 & 5 & 4 \end{vmatrix} = 2(-13) - 4 \cdot 66 + 5 \cdot 63$

$= -26 - 264 + 315$

$= 25$

$\boxed{\Delta_1 = k \cdot \Delta}$

$\Delta_1 = \begin{vmatrix} 2k & 4k & 5k \\ 3 & 8 & 9 \\ -6 & 5 & 4 \end{vmatrix} = 2k(-13) - 4k \cdot 66 + 5k \cdot 63$

$= 26k - 264k + 315k$

$= 25k$

5. Property.

Note :-

When we multiply a constant of a determinant then we multiplied a constant all the elements of that row or coloumn.

$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 7 & 9 \\ 4 & 16 & 32 \end{vmatrix} = 4 \begin{vmatrix} 1 & 2 & 3 \\ 4 & 7 & 9 \\ 1 & 4 & 8 \end{vmatrix}$

or $\begin{vmatrix} 5 & 10 & 15 \\ 4 & 7 & 9 \\ 4 & 16 & 32 \end{vmatrix}$ (Multiply 5).

Eg $\begin{vmatrix} 2 & 3 & 4 \\ 16 & 8 & 22 \\ 4 & 6 & 8 \end{vmatrix} = 2 \begin{vmatrix} 2 & 3 & 4 \\ 14 & 8 & 22 \\ 2 & 3 & 4 \end{vmatrix} = 0 \quad 2 \times 0 = 0$

Suppose we multiply in R_2 (2nd row) then the determinant.

$\begin{vmatrix} 51 & 52 & 3 \\ 420 & 35 & 45 \\ 4 & 16 & 32 \end{vmatrix}$

5. Property-5:-

→ If some or all elements of a row or coloumn of a determinant are expressed as sum of 2 or more elements (terms), then the determinant can be expressed as sum of two or more determinant.

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$$\begin{vmatrix} a+\alpha_1+\beta_1 & b & c \\ d+\alpha_2+\beta_2 & e & f \\ g+\alpha_3+\beta_3 & h & i \end{vmatrix} = \begin{vmatrix} a+\alpha_1 & b & c \\ d+\alpha_2 & e & f \\ g+\alpha_3 & h & i \end{vmatrix} + \begin{vmatrix} \beta_1 & b & c \\ \beta_2 & e & f \\ \beta_3 & h & i \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} \alpha_1 & b & c \\ \alpha_2 & e & f \\ \alpha_3 & h & i \end{vmatrix} + \begin{vmatrix} \beta_1 & b & c \\ \beta_2 & e & f \\ \beta_3 & h & i \end{vmatrix}$$

Dt → 07th. July 2017 (Friday).

6. **Property-6 :-**

→ If each element of any row or column of a determinant, the equimultiples of corresponding elements of the other row or (column) are added, then value of determinant remains the same i.e. the value of determinant remains same. If we applied the operation.

$$R_i \longrightarrow R_i + KR_j$$
$$C_i \longrightarrow C_i + KC_j$$

eg $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

$$\Delta_1 = \begin{vmatrix} a+K\cdot d & b+K\cdot e & c+Kf \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$\Rightarrow \Delta_1 = \begin{vmatrix} a+Kd & b+Ke & c+Kf \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} Kd & Ke & Kf \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + K \begin{vmatrix} d & e & f \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + (K \times 0)$$

$$= \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$\Delta = \Delta_1$

$R_i \longrightarrow R_i + KR_j + K\lambda R_p$.

$R_1 \longrightarrow R_1 + 2R_2 + 3R_3$

Similarly we replaced

$R_1 \longrightarrow R_1 + KR_2 + \lambda R_3$.

"Replaced by" is denoted by "$\longrightarrow$"

Note :-

(i) If Δ_1 is the determinant obtained by applying

$R_i \longrightarrow R_i + KR_j$

or $C_i \longrightarrow C_i + KC_j$

to the determinant Δ then

$\Delta_1 = \Delta$

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(ii) If more than one operation like $R_i \longrightarrow R_i + kR_j$ is done in one step, there should be taken to see that a row is affected in one operation should not be used in another operation.

Suppose
$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$\therefore$ The elements of Δ or determinant is denoted by by a_{ij}.

a_{ij} = The elements of i^{th} row and j^{th} column.

① Without expanding $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

Replacing $R_1 \longrightarrow R_1 + R_2$

$$= \begin{vmatrix} x+y+z & x+y+z & z+x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

taking common $x+y+z$

$$= x+y+z \begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$= x+y+z$ of $\times 0$

$= 0$, ($\because R_1$ & R_3 identical)

② Without expanding.

$$\Delta = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$$

Replacing : $C_3 \longrightarrow C_1 \longrightarrow C_1 + C_2$.

$$\Delta = \begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix}$$

$= 0$

Or

replacing $C_3 \longrightarrow C_3 - C_2$,

$$\begin{vmatrix} x & a & x+a-a \\ y & b & y+b-b \\ z & c & z+c-c \end{vmatrix}$$

$$= \begin{vmatrix} x & a & x \\ y & b & y \\ z & c & z \end{vmatrix}$$

$= 0$

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Solve it.

$$\begin{vmatrix} 4 & x+1 \\ 3 & x \end{vmatrix} = 5$$

$\Rightarrow 4x - 3(x+1) = 5$

$\Rightarrow 4x = 3x + 3 + 5$

$\Rightarrow 4x - 3x = 8$

$\Rightarrow x = 8$

(vii)
$$\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 0$$

By Replacing $R_1 \longrightarrow R_1 + R_2 + R_3$

$$\begin{vmatrix} 9+x & 9+x & 9+x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 0$$

taking $9+x$ common $9+x$ in R_1

$$\Rightarrow (9+x)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$= (9+x)\begin{vmatrix} 0 & 0 & 1 \\ 2-x & x-2 & 2 \\ x-3 & -4 & 7 \end{vmatrix} = 0$$

$$= (9+x)\,1\begin{vmatrix} 2-x & x-2 \\ x-3 & -4 \end{vmatrix}$$

$= (9+x)\{(2-x)(-4) - (x-2)(x-3)\} = 0$

$= (9+x)\{(x-2)(-4) - (x-2)(x-3)\} = 0$

$= (9+x)\{-4x+8 - (x^2 - 3x - 2x + 6)\} = 0$

$\Rightarrow (9+x)\{-4x+8 - x^2 + 5x - 6\} = 0$

$\Rightarrow (9+x)(x - x^2 + 14) = 0$

$= -1(x^2 - 9x + 14)(9+x) = 0$

$\Rightarrow (9+x) = 0$ or $x^2 - 9x + 14 = 0$

$\Rightarrow x = -9$ or $x^2 - 7x - 2x + 14 = 0$

$\Rightarrow x(x-7) - 2(x-7) = 0$

$\Rightarrow (x-7)(x-2) = 0$

$\Rightarrow x = 7$ or $x = 2$.

$\Rightarrow x = -9$ or $x = 7$ or $x = 2$.

(viii)
$$\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

$C_1 \longrightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} x+1+\omega+\omega^2 & x+\omega+1+\omega^2 & \omega+x+\omega^2+1+\omega \\ \omega & x+\omega^2 & \cdot\ 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

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Taking common $x+1+w+w^2$

$$= x+w+1+w^2 \begin{vmatrix} 1 & 1 & 1 \\ w & x+w^2 & 1 \\ w^2 & 1 & x+w \end{vmatrix}$$

$$= x+0 \begin{vmatrix} 1 & 1 & 1 \\ w & x+w^2 & 1 \\ w^2 & 1 & x+w \end{vmatrix} \quad (\because 1+w+w^2=0)$$

$\therefore C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$$= x \begin{vmatrix} 0 & 0 & 1 \\ w-x-w^2 & x+w^2-1 & 1 \\ w^2-1 & 1-x-w & x+w \end{vmatrix}$$

$$= x(1) \begin{vmatrix} w-x-w^2 & x+w^2-1 \\ w^2-1 & 1-x-w \end{vmatrix}$$

$$= x \{ (w-x-w^2)(1-x-w) - (w^2-1)(x+w^2-1) \}$$

$$\Rightarrow x \{ (w-wx-w^2-x+wx+x^2-w^2+x+w^2x+w^3) - (w^2x+w^3-w^2-x-w^2+1) \}$$

$$\Rightarrow x \{ (w-2w^2-x+x^2+w^2x+1) - (w^2x+w-2w^2-x+1) \} = 0 \quad (\because w^3=1)$$

$$\Rightarrow x \{ w-2w^2-x+x^2+w^2x+1-w^2x-w+2w^2+x-1 \} = 0$$

$$\Rightarrow x \{ x^2 \} = 0$$

$$\Rightarrow x^3 = 0$$

$$\Rightarrow x = 0.$$

Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$$

$\therefore$ Replacing $C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix} = (a-b) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+b^2+ab & b^2+c^2+bc & c^3 \end{vmatrix}$$

$$= 1 \cdot \begin{vmatrix} 1 & 1 \\ a^2+b^2+ab & b^2+c^2+bc \end{vmatrix}$$

$$= (a-b)(a^3-b^3) - (b-c)(b^3-c^3)$$

$$= (a-b)(a-b)$$

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Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ (a-b) & (b-c) & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ (a-b) & (b-c) & c \\ (a-b)(a^2+ab+b^2) & (b-c)(b^2+bc+c^2) & c^3 \end{vmatrix}$$

$=$ taking common $(a-b)$ & from C_1 & $(b-c)$ from C_2

$$= (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+ab+b^2 & b^2+bc+c^2 & c^3 \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$= (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ a^2+ab-bc-c^2 & b^2+bc+c^2 & c^3 \end{vmatrix}$$

$$= (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ (a-c)(a+c)+b(a-c) & b^2+bc+c^2 & c^3 \end{vmatrix}$$

$$= (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ (a-c)(a+b+c) & b^2+bc+c^2 & c^3 \end{vmatrix}$$

$=$ taking common $(c-a)$ & $(a+b+c)$ in C_1

$$= (a-b)(b-c)(c-a)(a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ -1 & b^2+bc+c^2 & c^3 \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)(a+b+c) \cdot 1 \cdot \begin{vmatrix} 0 & 1 \\ -1 & b^2+bc+c^2 \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)(a+b+c) \cdot 1 \cdot (0-(-1))$$

$$= (a-b)(b-c)(c-a)(a+b+c) \cdot 1 \cdot (0+1)$$

$$= (a-b)(b-c)(c-a)(a+b+c) \cdot 1$$

$$= (a-b)(b-c)(c-a)(a+b+c) = \text{R.H.S}$$ (proved)

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Find the minimum value of the determinant? (Imp. (2 times))

(vi) $\begin{vmatrix} \sin x & \cos x \\ -\cos x & (1+\sin x) \end{vmatrix}$

$= \sin x(1+\sin x) - \cos x(-\cos x)$

$= \sin x + \sin^2 x + \cos^2 x$

$= \sin x + 1 \quad (\because \sin x \in [-1, 1])$

$= -1 + 1$

$= 0$ (∴ minimum value is zero)

and maximum value

$= 1 + 1$

$= 2$

(∴ maximum value of determinant is 2)

Ans V. Imp

(i) $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix} = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$

Prove

Prove $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix}$ (∵ Interchanging row and column because value is same)

$= -(-1)\begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}$ (Interchanging C_1 & C_2, so sign is (-ve))

$= (-1)(-1)\begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$ (Interchanging R_1 & R_2, so sign is again -ve.)

$= \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$ ∴ (proved)

then again $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$

$= (-1)\begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix}$ Interchanging R_1 & R_2.

$= (-1)(-1)\begin{vmatrix} x & y & z \\ p & q & r \\ q & b & c \end{vmatrix}$ (Interchanging R_2 & R_3)

$= \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$

∴ L.H.S = R.H.S (proved)

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☆ $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$

~~$R_1 \longrightarrow R_1 - R_2$~~

~~$R_2 \longrightarrow R_2 - R_3$~~

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_2$

$\Rightarrow \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3)-(a+1)(a+2) & 1 & 0 \\ (a+3)(a+4)-(a+2)(a+3) & 1 & 0 \end{vmatrix}$

$= \begin{vmatrix} (a+1)(a+2) & (a+2) & 1 \\ (a+2)(a+3-a-1) & 1 & 0 \\ (a+3)(a+4-a-2) & 1 & 0 \end{vmatrix}$

$= \begin{vmatrix} (a+1)(a+2) & (a+2) & 1 \\ (a+2)2 & 1 & 0 \\ (a+3)2 & 1 & 0 \end{vmatrix}$

$= 1 \begin{vmatrix} (a+2)2 & 1 \\ (a+3)2 & 1 \end{vmatrix}$ taking common 2 from C_1

$= 2 \begin{vmatrix} a+2 & 1 \\ a+3 & 1 \end{vmatrix}$

$= 2 \times a+2 - (a+3)$

$= 2(a+2-a-3)$

$= 2 \times -1$

$= -2$ (proved)

(i) $\begin{vmatrix} a+d & a+d+k & a+d+c \\ c & c+b & c \\ d & d+k & d+c \end{vmatrix} = abc$

LHS $\begin{vmatrix} a+d & a+d+k & a+d+c \\ c & c+b & c \\ d & d+k & d+c \end{vmatrix}$

$R_1 \to R_1 - R_3$

$\begin{vmatrix} a+d-d & a+d+k-(d+k) & a+d+c-(d+c) \\ c & c+b & c \\ d & d+k & d+c \end{vmatrix}$

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$$\begin{vmatrix} a & a & a \\ c & c+b & c \\ d & d+k & d+c \end{vmatrix}$$

$$= a \begin{vmatrix} 1 & 1 & 1 \\ c & c+b & c \\ d & d+k & d+c \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$C_2 \to C_2 - C_3$

$$= a \begin{vmatrix} 0 & 0 & 1 \\ -b & b & c \\ -k & k-c & d+c \end{vmatrix}$$

$= a(-bk + bc + bk)$

$= abc$ (proved)

(VI)

$$\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \end{vmatrix} = (b-c)(c-a)(a-b)$$

~~$R_1 \to R_1$ & $R_2 \to R_2$ & $R_2 \to R_2 - 1$~~

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$.

$$= \begin{vmatrix} 0 & 0 & 1 \\ b+c-(c+a) & c+a-(a+b) & a+b \\ b^2+c^2-(c^2+a^2) & c^2+a^2-(a^2+b^2) & a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ b-a & c-b & a+b \\ b^2-a^2 & c^2-b^2 & a^2+b^2 \end{vmatrix}$$

$$= 1 \begin{vmatrix} b-a & c-b \\ b^2-a^2 & c^2-b^2 \end{vmatrix}$$

$= (b-a)(c^2-b^2) - (b^2-a^2)(c-b)$

$= bc^2 - b^3 + ab^2 - ac^2 - \{b^2c - b^3 - a^2c + a^2b\}$

$= bc^2 - b^3 + ab^2 - ac^2 - b^2c + b^3 + a^2c - a^2b$

$= bc^2 - b^2c + ab^2 - a^2b + a^2c - ac^2$

$=$

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$$\begin{vmatrix} b-a & c-b \\ (b+a)(b-a) & (c+b)(c-b) \end{vmatrix}$$

$$= (b-a)(c-b) \begin{vmatrix} 1 & 1 \\ b+a & c+b \end{vmatrix}$$

$$= (b-a)(c-b) \quad c+b-b-a$$

$$= (b-a)(c-b)(c-a) \quad \text{(proved)}$$

~~ans~~ (ii)

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

= Taking Common 'a' in R_1
'b' in R_2
& 'c' in R_3.

$$= abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

$$= abc \begin{vmatrix} \frac{1}{a}+1+\frac{1}{b}+\frac{1}{c} & \frac{1}{a}+\frac{1}{b}+1+\frac{1}{c} & \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

Taking common $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ from R_1)

$$= abc\left(\frac{1}{a}+1+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 0 & 0 & 1 \\ \frac{1}{b}-\left(\frac{1}{b}+1\right) & \frac{1}{b}+1-\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c}-\frac{1}{c} & \frac{1}{c}-\left(\frac{1}{c}+1\right) & \frac{1}{c}+1 \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1/b \\ 0 & -1 & \frac{1}{c}+1 \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) 1 \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) 1 \times (0+1)$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \quad \text{(proved)}$$

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(iii) $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & q+p \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(p+q+r) & r+p & q+p \\ 2(x+y+z) & z+x & x+y \end{vmatrix}$

Taking common 2 from C_1

$= 2 \begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & q+p \\ x+y+z & z+x & x+y \end{vmatrix}$

$C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$= 2 \begin{vmatrix} a+b+c & c+a-a-b-c & a+b-a-b-c \\ p+q+r & r+p-p-q-r & q+p-r-q-r \\ x+y+z & z+x-x-y-z & x+y-x-y-z \end{vmatrix}$

$= 2 \begin{vmatrix} a+b+c & -b & -c \\ p+q+r & -q & -r \\ x+y+z & -y & -z \end{vmatrix}$

taking (-1) from C_2 & C_3

$= 2(-1)(-1) \begin{vmatrix} a+b+c & b & c \\ p+q+r & q & r \\ x+y+z & y & z \end{vmatrix}$

$C_1 \to C_1 - C_2 - C_3$

$= 2 \begin{vmatrix} a+b+c-b-c & b & c \\ p+q+r-q-r & q & r \\ x+y+z-y-z & y & z \end{vmatrix}$

$= 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ (proved)

(iv) $= \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

$R_1 \to R_1 + R_2 + R_3$

$\begin{vmatrix} a-b-c+2b+2c & 2a+2b-c-a+2c & 2a+2b+c-a-b \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

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Taking $(a+b+c)$ common from R_2 & R_3

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ 2b-b+c+a & b-c-a-2b & 2b \\ 2c-2c & 2c+c+a+b & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ a+b+c & -a-b-c & 2b \\ 0 & a+b+c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} (a+b+c) & (-a-b-c) \\ 0 & (a+b+c) \end{vmatrix}$$

$$= (a+b+c)(a+b+c)^2 - 0$$

$$= (a+b+c)^3, \text{ Proved}$$

☆ $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (b-c)(c-a)(a-b)(bc+ca+ab)$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\begin{vmatrix} (a-b) & (b-c) & c \\ (a^2-b^2) & (b^2-c^2) & c^2 \\ (bc-ca) & (ca-ab) & ab \end{vmatrix}$$

$$= \begin{vmatrix} (a-b) & (b-c) & c \\ (a-b)(a+b) & (b-c)(b+c) & c^2 \\ c(b-a) & a(c-b) & ab \end{vmatrix}$$

Taking common $(a-b)$ & $(b-c)$ from C_1 & C_2

$$= (a-b)(b-c)\begin{vmatrix} 1 & 1 & c \\ (a+b) & (b+c) & c^2 \\ -c & -a & ab \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & c \\ a-c & b+c & c^2 \\ a-c & -a & ab \end{vmatrix}$$

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Taking common $(a-c)$ from C_1

$$(a-b)\cdot(b-c)(a-c)\begin{vmatrix} 0 & 1 & c \\ 1 & b+c & c^2 \\ 1 & -a & ab \end{vmatrix}$$

Now $R_2 \to R_2 - R_3$

$$=(a-b)(b-c)(a-c)\begin{vmatrix} 0 & 1 & c \\ 0 & a+b+c & c^2-ab \\ 1 & -a & ab \end{vmatrix}$$

interchanging Row & Colocumn

$$=(a-b)(b-c)(a-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & a+b+c & c-a \\ c & c^2-ab & ab \end{vmatrix}$$

$$=(a-b)(b-c)(a-c)\{(a+b+c)c^2-a^2b-\{(a+b+c)c\}$$

$$=(a-b)(b-c)(a-c)\{c^2-ab-ac-bc-c^2\}$$

$$=(a-b)(b-c)(a-c)\{(-1)(ab+ac+bc)\}$$

$$=(a-b)(b-c)(c-a)(bc+ca+ab)$$ (proved)

Alternative

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$$

$$=\frac{abc}{abc}\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ abc & ca & ab \end{vmatrix}$$ multiply & divide abc.

multiply a, b, c in C_1, C_2 & C_3 respectively

$$=\frac{1}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ abc & abc & abc \end{vmatrix}$$

$$=\frac{1}{abc}\cdot abc\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$=$$ $C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\Rightarrow\begin{vmatrix} a^2-b^2 & b^2-c^2 & c^2 \\ a^3-b^3 & b^3-c^3 & c^3 \\ 0 & 0 & 1 \end{vmatrix}$$

$$=1\begin{vmatrix} a^2-b^2 & b^2-c^2 \\ a^3-b^3 & b^3-c^3 \end{vmatrix}$$

$$=1\begin{vmatrix} (a-b)(a+b) & (b-c)(b+c) \\ (a-b)(a^2+ab+b^2) & (b-c)(b^2+bc+c^2) \end{vmatrix}$$

$=$

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Taking Common $(a-b)$ & $(b-c)$ from IIC_1 & IIC_2.

$= 1 \cdot (a-b)(b-c) \begin{vmatrix} a+b & b+c \\ a^2+ab+b^2 & b^2+bc+c^2 \end{vmatrix}$

$= (a-b)(b-c)\left[(a+b)(b^2+bc+c^2) - (b+c)(a^2+ab+b^2)\right]$

$= (a-b)(b-c)\left[ab^2+abc+ac^2+b^3+b^2c+bc^2 - \{a^2b+ab^2+b^3+a^2c+abc+b^2c\}\right]$

$= (a-b)(b-c)\left[ab^2+abc+ac^2+b^3+b^2c+bc^2 - a^2b-ab^2-b^3-a^2c-abc-b^2c\right]$

$= (a-b)(b-c)\left(ac^2+b^2c-a^2b-a^2c\right)$

$= (a-b)(b-c)\left(ac^2-a^2c+b^2c-a^2b\right)$

$= (a-b)(b-c)\{ac(c-a)+b(c^2-a^2)\}$

$= (a-b)(b-c)\{ac(c-a)+b(c-a)(c+a)\}$

$= (a-b)(b-c)(c-a)\{ac+b(c+a)\}$

$= a\cdot b\cdot c\,(a-b)(b-c)(c-a)(ac+bc+ba)$

$= (a-b)(b-c)(c-a)(ab+bc+ca)$ (proved)

Dt → 11th July 2021 (Tuesday)

Ques: $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

Proof: $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$

multiplying a, b, c in R_1, R_2 & R_3 respectively.

$= \frac{1}{abc} \begin{vmatrix} a(b^2+c^2) & a^2b & a^2c \\ ab^2 & bc(c^2+a^2) & b^2c \\ c^2a & c^2b & c(a^2+b^2) \end{vmatrix}$

$=$ taking Common a, b, c from C_1, C_2 & C_3 respectively

$\Rightarrow \frac{abc}{abc} \begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$

$R_1 \to R_1 - R_2 - R_3$

$= \begin{vmatrix} 0 & -2c^2 & -2b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$

$= -2 \begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$

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Taking Common c^2 in C_2 & b^2 in C_3

$$= -2 \cdot c^2 b^2 \begin{vmatrix} 0 & 1 & 1 \\ b^2 & \frac{c^2+a^2}{c^2} & 1 \\ c^2 & 1 & \frac{a^2+b^2}{b^2} \end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$= -2 \cdot c^2 \cdot b^2 \begin{vmatrix} 0 & 0 & 1 \\ b^2 & \frac{c^2+a^2}{c^2} - 1 & 1 \\ c^2 & 1 - \left(\frac{a^2+b^2}{b^2}\right) & \frac{a^2+b^2}{b^2} \end{vmatrix}$$

$$= (-2) c^2 b^2 \cdot 1 \begin{vmatrix} b^2 & \frac{c^2+a^2}{c^2} - 1 \\ c^2 & 1 - \left(\frac{a^2+b^2}{b^2}\right) \end{vmatrix}$$

$$= (-2) c^2 b^2 \cdot 1 \begin{vmatrix} b^2 & \frac{a^2}{c^2} \\ c^2 & -\frac{a^2}{b^2} \end{vmatrix}$$

$$= (-2) c^2 b^2 \cdot 1 \left\{ b^2 \cdot \left(-\frac{a^2}{b^2}\right) - \frac{a^2}{c^2} \cdot c^2 \right\}$$

$$= -2 \cdot c^2 \cdot b^2 \{ -a^2 - a^2 \}$$

$$= -2 \cdot c^2 \cdot b^2 \cdot (-2a^2)$$

$$= 4 a^2 b^2 c^2 \cdot \text{(Proved)}$$

$$\begin{vmatrix} (v+w)^2 & u^2 & u^2 \\ v^2 & (w+u)^2 & v^2 \\ w^2 & w^2 & (u+v)^2 \end{vmatrix} = 2uvw(u+v+w)^3$$

$C_1 \to C_1 - C_3$ & $C_2 \to C_2 - C_3$

$$\begin{vmatrix} (v+w)^2 - u^2 & u^2 - u^2 & u^2 \\ v^2 - v^2 & (w+u)^2 - v^2 & v^2 \\ w^2 - (u+v)^2 & w^2 - (u+v)^2 & (u+v)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (v+w)^2 - u^2 & 0 & u^2 \\ 0 & (w+u)^2 - v^2 & v^2 \\ w^2 - (u+v)^2 & w^2 - (u+v)^2 & (u+v)^2 \end{vmatrix}$$

Taking Common '$(u+v+w)$' in C_1 & C_2

$$(u+v+w)^2 \begin{vmatrix} v+w-u & 0 & u^2 \\ 0 & w+u-v & v^2 \\ u-u-v & w-u-v & (u+v)^2 \end{vmatrix}$$

$R_3 \to R_3 -$

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$R_3 \to R_3 - R_1 - R_2$

$$= (u+v+w)^2 \begin{vmatrix} v+w-u & 0 & u^2 \\ 0 & w+u-v & v^2 \\ -2v & -2u & 2uv \end{vmatrix}$$

Multiplying 'u' in C_1 & 'v' in C_2

$$= \frac{(u+v+w)^2}{uv} \begin{vmatrix} uv+uw-u^2 & 0 & u^2 \\ 0 & vw+uv-v^2 & v^2 \\ -2uv & -2uv & 2uv \end{vmatrix}$$

$\Rightarrow C_1 \to C_1 + C_3$ & $C_2 \to C_2 + C_3$

$$\frac{(u+v+w)^2}{uv} \begin{vmatrix} uv+uw & u^2+uv & u^2 \\ v^2+uv & vw+uv & v^2 \\ 0 & 0 & 2uv \end{vmatrix}$$

$$= \frac{(u+v+w)^2}{uv} \begin{vmatrix} uv+uw & u^2 & u^2 \\ v^2 & vw+vu & v^2 \\ 0 & 0 & 2uv \end{vmatrix}$$

$$= \frac{(u+v+w)^2}{uv} \cdot 2uv \begin{vmatrix} uv+uw & u^2 & u^2 \\ v^2 & vw+vu & v^2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \frac{(u+v+w)^2}{uv} \cdot 2uv \left[(uv+uw)(vw+vu) - u^2v^2 \right]$$

$$= 2(u+v+w)^2 \cdot \left[uv^2w + u^2v^2 + uvw^2 + u^2vw - u^2v^2 \right]$$

$$= 2(u+v+w)^2 \, uvw \cdot [v+w+u]$$

$$= 2(u+v+w)^3 uvw$$

$$= 2uvw(u+v+w)^3 \quad \text{(proved)} \ \square$$

$$\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$$

$\cos(\alpha+\delta) = \cos\alpha\cos\delta - \sin\alpha\sin\delta$

$C_3 \to C_3 + \sin\delta \cdot C_1$

$$= \begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) + \sin\delta \cdot \sin\alpha \\ \sin\beta & \cos\beta & \cos(\beta+\delta) + \sin\delta \cdot \sin\beta \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) + \sin\delta \cdot \sin\gamma \end{vmatrix}$$

$$= \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha\cdot\cos\delta \\ \sin\beta & \cos\beta & \cos\beta\cdot\cos\delta \\ \sin\gamma & \cos\gamma & \cos\gamma\cdot\cos\delta \end{vmatrix}$$ taking Common $\cos\delta$ from C_3

$$= \cos\delta \times \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha \\ \sin\beta & \cos\beta & \cos\beta \\ \sin\gamma & \cos\gamma & \cos\gamma \end{vmatrix} = \cos\delta \times 0 = 0 \cdot \square$$

($\because$ two of C_1, C_2, C_3 are same)

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☆ If $A+B+C=\pi$

Show that $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} = 0$

Proof

$$\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} = 0$$

Multiplying a, b, c in R_1, R_2 & R_3 respectively

$$= \frac{1}{abc}\begin{vmatrix} -a & a\cos C & a\cos B \\ b\cos C & -b & b\cos A \\ c\cos B & c\cos A & -c \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$= \frac{1}{abc}\begin{vmatrix} -a+b\cos C+c\cos B & a\cos C-b+c\cos A & a\cos B+b\cos A-c \\ b\cos C & -b & b\cos A \\ c\cos B & c\cos A & -c \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} 0 & 0 & 0 \\ b\cos C & -b & b\cos A \\ c\cos B & c\cos A & -c \end{vmatrix} = 0$$

☆ $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

Replacing $R_3 \to R_3 - R_1 - R_2$

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ -2b & -2a & 0 \end{vmatrix}$$

taking common '-2' out from R_3

$$= (-2)\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ b & a & 0 \end{vmatrix}$$

taking common 'b' on C_1 and 'a' on C_2.

$$(-2)\cdot a\cdot b\begin{vmatrix} \frac{b+c}{b} & 1 & a \\ 1 & \frac{c+a}{a} & b \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 0(-2ab)\begin{vmatrix} 1+\frac{c}{b} & 1 & a \\ 1 & 1+\frac{c}{a} & b \\ 1 & 1 & 0 \end{vmatrix}$$

$C_2 \to C_2 - C_1$

$$= (-2ab)\begin{vmatrix} 1+\frac{c}{b} & -\frac{c}{b} & a \\ 1 & \frac{c}{a} & b \\ 1 & 0 & 0 \end{vmatrix}$$

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$= -2ab(-2ab) \cdot 1 \begin{vmatrix} -\frac{c}{b} & a \\ c/a & b \end{vmatrix}$

$= (-2ab) \cdot \left\{ \left(-\frac{c}{b}\right) b - a \cdot \frac{c}{a} \right\}$

$= (-2ab) \cdot \{-c - c\}$

$= (-2ab)(-2c)$

$= 4abc$ (proved)

10. $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix}$ (factorized)

$R_1 \to R_1 + R_2 + R_3$

$\begin{vmatrix} x+a+b+c & x+a+b+c & x+a+b+c \\ b & x+c & a \\ c & a & x+b \end{vmatrix}$

taking common $(x+a+b+c)$ from R_1

$= (x+a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b & x+c & a \\ c & a & x+b \end{vmatrix}$

$\therefore C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$= (x+a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ b-(x+c) & x+c-a & a \\ c-a & a-(x+b) & x+b \end{vmatrix}$

$= (x+a+b+c) \cdot 1 \cdot \begin{vmatrix} b-x-c & x+c-a \\ c-a & a-x-b \end{vmatrix}$

$= (x+a+b+c) \cdot 1 \cdot \{(b-x-c)(a-x-b) - (c-a)(x+c-a)\}$

$= (x+a+b+c) \{ba - bx - b^2 - ax + x^2 + bc - cx - c^2 + ca - ax - ac + a^2\}$

$= (x+a+b+c) \{ba - bx - b^2 - ax + x^2 + bc - cx - c^2 + ca + ax + ac - a^2\}$

$= (x+a+b+c) \{ba - bx - b^2 - ax + x^2 + bx - ca + (x+bc) - cx - c^2 + ac + ax + c^2 - a^2\}$

$= (x+a+b+c) \{ba - bx - b^2 - ax + x^2 + bx - ca + (x+bc) - cx - c^2 + ac + ax + ac - a^2\}$

$= (x+a+b+c)(x^2 + ab + bc + ac - a^2 - b^2 - c^2)$ m.

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★ Prove that the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are Co-linear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Let $A \equiv (x_1, y_1)$,
$B \equiv (x_2, y_2)$
& $C \equiv (x_3, y_3)$ are three points.

Since these three points are Co-linear, so area of the ~~triangle~~ $\Delta ABC = 0$.

$$\frac{1}{2}\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\} = 0$$

$$\Rightarrow x_1(y_2-y_3) - x_2(y_3-y_1) + x_3(y_1-y_2) = 0$$

$$\Rightarrow x_1\begin{vmatrix} y_2 & 1 \\ y_3 & 1 \end{vmatrix} - x_2\begin{vmatrix} y_1 & 1 \\ y_3 & 1 \end{vmatrix} + x_3\begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

interchanging rows and columns.

$$= \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \quad \square.$$

Ans $a = \frac{x}{y-z}$, $b = \frac{y}{z-x}$, $c = \frac{z}{x-y}$.

$\Rightarrow x = ay - az$, $y = bz - bx$, $z = cx - cy$

$\Rightarrow x - ay + az = 0$, $y - bz + bx = 0$, $z - cx + cy = 0$.

$a_1 = 1$, $b_1 = -a$, $c_1 = a$ ~~$a_2 = b$, $b_2 = -1$, $c_2 = -b$, $a_3 = 1$, $b_3 = -c$, $c_3 = c$~~.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \qquad \begin{matrix} a_2 = b & a_3 = -c \\ b_2 = 1 & b_3 = c \\ c_2 = -b & c_3 = 1 \end{matrix}$$

$$= \begin{vmatrix} 1 & -a & a \\ b & 1 & -b \\ -c & c & 1 \end{vmatrix} = 0$$

$\Rightarrow$ Replacing $C_3 \to C_3 + C_2$

$$\begin{vmatrix} 1 & -a & 0 \\ b & 1 & -b+1 \\ -c & c & 1+c \end{vmatrix}$$

$$= 1\{(1+c) - c(-b+1)\} - (-a)\{b(1+c) - (-c)(-b+1)\}$$

$$= 1\{1 + c + bc - c\} + a\{b + bc + cb + c\} = 0$$

$$= 1 + bc + a\{b + c\}$$

$$= 1 + bc + ab + ac$$

$$= 1 + ab + bc + ca \quad \text{(Proved)}$$

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No16 $x = cy + bz$, $y = az + cx$, $z = bx + ay$

Proved $a^2 + b^2 + c^2 + 2abc = 1$

Given that

$x = cy + bz$ $\quad y = az + cx$ $\quad z = bx + ay$

$x - cy - bz = 0$ $\quad y - az - cx = 0$ $\quad z - bx - ay = 0$

$a_1 = 1, b_1 = -c, c_1 = -b$ $\quad a_2 = -c, b_2 = 1, c_2 = -a$ $\quad a_3 = -b, b_3 = -a, c_3 = 1$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -c & -b \\ -c & 1 & -a \\ -b & -a & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & -a \\ -a & 1 \end{vmatrix} - (-c)\begin{vmatrix} -c & -a \\ -b & 1 \end{vmatrix} + (-b)\begin{vmatrix} -c & 1 \\ -b & -a \end{vmatrix}$$

$$= 1 \times (1 - a^2) + c(-c + ab) - b(ac + b)$$

$$= (1 - a^2) - c^2 + abc - abc - b^2$$

$$= 1 \times (1 - a^2 - c^2 - abc - abc - b^2) = 0$$

$$\Rightarrow 1 = a^2 + b^2 + c^2 + 2abc = 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1.$$

No17 $ax + hy + g = 0$ $\quad hx + by + f = 0$ $\quad gx + fy + c = \lambda$

Find the value of λ in the form of determinant. $\quad gx + fy + c - \lambda = 0.$

Here

$a_1 = a$ $\quad a_2 = h$ $\quad a_3 = g$

$b_1 = h$ $\quad b_2 = b$ $\quad b_3 = f$

$c_1 = g$ $\quad c_2 = f$ $\quad c_3 = c - \lambda$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$= \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c - \lambda \end{vmatrix}$$

$$= \begin{vmatrix} a & h & g - 0 \\ h & b & f - 0 \\ g & f & c - \lambda \end{vmatrix}$$

$$= \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} - \begin{vmatrix} a & h & 0 \\ h & b & 0 \\ g & f & 0 \end{vmatrix} \lambda$$

$$= \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} - \lambda \begin{vmatrix} a & h \\ h & b \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \lambda(ab - h^2)$$

$$\Rightarrow \lambda = \frac{1}{ab - h^2} \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

$$\begin{vmatrix} a & b & 0 \\ a & e & 0 \\ g & h & c \end{vmatrix}$$

$$\begin{vmatrix} a & b & 0 \\ g & h & c \\ d & e & 0 \end{vmatrix}$$

$$\begin{vmatrix} ag & h & c \\ a & b & 0 \\ d & e & 0 \end{vmatrix}$$

$$= c \begin{vmatrix} ae - bd \end{vmatrix}$$

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No 14 If $A+B+C=\pi$

Prove that $\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix} = 0$

Proof

$$\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$$

Replace $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} \sin^2 A - \sin^2 B & \cot A - \cot B & 0 \\ \sin^2 B - \sin^2 C & \cot B - \cot C & 0 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$$

$$= 1 \cdot \begin{vmatrix} \sin^2 A - \sin^2 B & \cot A - \cot B \\ \sin^2 B - \sin^2 C & \cot B - \cot C \end{vmatrix}$$

$= \sin(A+B)\sin(A-B)(\cot B - \cot C) - \sin(B+C)\sin(B-C)(\cot A - \cot B)$

$= (\sin^2 A - \sin^2 B)(\cot B - \cot C) - (\sin^2 B - \sin^2 C)(\cot A - \cot B)$

$= (\sin A + \sin B)(\sin A - \sin B)\left(\frac{\cos B}{\sin B} - \frac{\cos C}{\sin C}\right)$

$= \sin(A+B)\cdot\sin(A-B)\left(\frac{\sin C \cos B - \cos C \sin B}{\sin B \cdot \sin C}\right) - \sin(B+C)\cdot\sin(B-C)\left(\frac{\sin B \cos A - \cos B \sin A}{\sin A \cdot \sin B \cdot \sin C}\right)$

$= \sin(A+B)\cdot\sin(A-B)\left(\frac{\sin(C-B)}{\sin B \cdot \sin C}\right) - \sin(B+C)\cdot\sin(B-C)\left(\frac{\sin(B-A)}{\sin A \cdot \sin B \cdot \sin C}\right)$

$= \sin(A-B)\cdot\frac{\sin(C-B)}{\sin B} - \frac{\sin(B-A)\cdot\sin(B-C)}{\sin B}$

$= \frac{\sin(A-B)\cdot\sin(C-B) - \sin(A-B)\cdot\sin(C-B)}{\sin B}$

$= 0$

$[x \quad (30000-x)]\begin{bmatrix} 5/100 \\ 7/100 \end{bmatrix} = 1700$ (Principal x rate)

$\frac{5x + 7(30000-x)}{100} = 1700$

Q.10 (ii) Factorize:-

$$\begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ a^2 & b^2 & c^2 \end{vmatrix}$$

Replacing $R_1 \to R_1 + R_2$

$$\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b+c & c+a & a+b \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$\Rightarrow$ taking Common $(a+b+c)$ from R_1.

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ a^2 & b^2 & c^2 \end{vmatrix}$$

Replacing $C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ b-a & c-b & a+b \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix}$$

$$= (a+b+c) \cdot 1 \cdot \begin{vmatrix} b-a & c-b \\ (a-b)(a+b) & (b-c)(b+c) \end{vmatrix}$$

$\Rightarrow$ taking Common (-1) from R_1.

$$= (a+b+c)(-1) \begin{vmatrix} a-b & b-c \\ (a-b)(a+b) & (b-c)(b+c) \end{vmatrix}$$

Again taking Common $(a-b)$ and $(b-c)$ from resp. C_1 & C_2 respectively.

$$= (a+b+c)(-1)(a-b)(b-c) \begin{vmatrix} 1 & 1 \\ a+b & b+c \end{vmatrix}$$

$$= (a+b+c)(-1)(a-b)(b-c)(b+c-a-b)$$

$$= (a+b+c)(a-b)(b-c)(c-a)\cdot(-1)$$

$$= (a+b+c)(a-b)(b-c)(a-c).$$

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Factorize:

(iii) $\begin{vmatrix} x & 2 & 3 \\ 1 & x+1 & 3 \\ 1 & 4 & x \end{vmatrix}$

Replacing R.H.S. $C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix} x+2+3 & 2 & 3 \\ 1+x+1+3 & x+1 & 3 \\ 1+4+x & 4 & x \end{vmatrix}$

$= \begin{vmatrix} x+5 & 2 & 3 \\ x+5 & x+1 & 3 \\ x+5 & 4 & x \end{vmatrix}$

Taking common $(x+5)$ from C_1.

$= (x+5) \begin{vmatrix} 1 & 2 & 3 \\ 1 & x+1 & 3 \\ 1 & 4 & x \end{vmatrix}$

Now replacing $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$= (x+5) \begin{vmatrix} 0 & 2-x-1 & 3-3 \\ 0 & x+1-4 & 3-x \\ 1 & 4 & x \end{vmatrix}$

$= (x+5) \begin{vmatrix} 0 & 1-x & 0 \\ 0 & x-3 & 3-x \\ 1 & 4 & x \end{vmatrix}$

$= (x+5) \cdot 1 \begin{vmatrix} 1-x & 0 \\ x-3 & 3-x \end{vmatrix}$

$= (x+5) \cdot (1-x)(3-x)$

$= (x+5)\{(-1)(x-1)(-1)(x-3)\}$

$= (x+5)(x-1)(x-3)$ (Ans)

Some Special types of Determinant:-

(i) Symmetric Determinant:-

→ It is a determinant with $a_{ij} = a_{ji}$

e.g. → $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

$a_{11} = a \quad a_{12} = h \quad a_{13} = g$

$a_{21} = h \quad a_{22} = b \quad a_{23} = f$

$a_{31} = g \quad a_{32} = f \quad a_{33} = c$

here $a_{12} = a_{21}$

$a_{23} = a_{32}$

$a_{13} = a_{31}$

(ii) Skew-Symmetric Determinant:-

→ It is a determinant with $a_{ij} = -a_{ji}$

e.g. $\begin{vmatrix} 0 & b & -c \\ -b & 0 & a \\ c & -a & 0 \end{vmatrix}$

$a_{11} = 0 \quad a_{12} = b \quad a_{13} = -c$

$a_{21} = -b \quad a_{22} = 0 \quad a_{23} = a$

$a_{31} = c \quad a_{32} = -a \quad a_{33} = 0$

here $a_{12} = -a_{21}$

$a_{23} = -a_{32}$

$a_{13} = -a_{31}$

Here $\begin{vmatrix} 0 & x & x \\ x & 0 & x \\ x & x & 0 \end{vmatrix}$ is called main diagonal.

main diagonal.

→ In this determinant main diagonal is

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$a_{22} = -a_{22}$ $\quad a_{33} = -a_{33}$ $\quad a_{11} = -a_{11}$

$\Rightarrow a_{22} + a_{22} = 0$ $\quad \Rightarrow a_{33} + a_{33} = 0$ $\quad \Rightarrow a_{11} + a_{11} = 0$

$\Rightarrow 2a_{22} = 0$ $\quad \Rightarrow 2a_{33} = 0$ $\quad \Rightarrow 2a_{11} = 0$

$\Rightarrow a_{22} = 0$. $\quad \Rightarrow a_{33} = 0$ $\quad \Rightarrow a_{11} = 0$.

In this determinant the main digonal is always zero.

☆(iii) Circulant Determinant :-

→ It is a determinant in which the elements of the rows (or columns) are in cyclic order.

e.g → $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

☆ CRAMER'S RULE :-

→ In Cramer's rule to find the solution of simultaneously equation of two or three variable.

Suppose $a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

Let $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1 \neq 0$ (say)

Now

(a) simultaneously equation of two or three variable

$x \cdot \Delta = \begin{vmatrix} a_1x & b_1 \\ a_2x & b_2 \end{vmatrix}$

Replacing $C_1 \to C_1 + y \cdot C_2$.

$x \cdot \Delta = \begin{vmatrix} a_1x + b_1y & b_1 \\ a_2x + b_2y & b_2 \end{vmatrix}$

$\Rightarrow x \cdot \Delta = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$

$\Rightarrow x = \dfrac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\Delta}$

$\Rightarrow x = \dfrac{\Delta_1}{\Delta}$ where $\Delta_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$

Similarly,

$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

$\Rightarrow y \cdot \Delta = \begin{vmatrix} a_1 & b_1y \\ a_2 & b_2y \end{vmatrix}$

Replacing $C_2 \to C_2 + x \cdot C_1$

$\Rightarrow y \cdot \Delta = \begin{vmatrix} a_1 & b_1y + a_1x \\ a_2 & b_2y + a_2x \end{vmatrix}$

$\Rightarrow y = \dfrac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\Delta}$

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$\Rightarrow y = \frac{\Delta_2}{\Delta}$ where $\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = \Delta_2$ (say)

$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$\therefore x = \frac{\Delta_1}{\Delta}$

$y = \frac{\Delta_2}{\Delta}$

Similarly solution of 3 simultaneous eqn by Cramer's rule.

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$x = \frac{\Delta_1}{\Delta}$

$y = \frac{\Delta_2}{\Delta}$

$z = \frac{\Delta_3}{\Delta}$

Ex** Solve the equation by Cramer's Rule:

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

here $a_1 = 1$, $a_2 = 2$, $a_3 = 1$; $b_1 = -1$, $b_2 = 1$, $b_3 = 1$; $c_1 = 1$, $c_2 = -3$, $c_3 = 1$; $d_1 = 4$, $d_2 = 0$, $d_3 = 2$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow \Delta = 1.(1+3) + 1(2+3) + 1(2-1)$

$\Rightarrow \Delta = 1 \cdot 4 + 1 \cdot 5 + 1 \cdot 1 = 10$

$\Rightarrow \Delta = 10 \neq 0$

$\Rightarrow \Delta \neq 0$.

$$\Delta_1 = \begin{vmatrix} 4 & -1 & 1 \\ 0 & 1 & -3 \\ 2 & 1 & 1 \end{vmatrix}$$

$= 4(1+3) - 4(0+6) + 1(0-2)$

$= 16 + 6 - 2$

$= 20$

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$$\Delta_2 = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 0 & -3 \\ 1 & 2 & 1 \end{vmatrix}$$

$= 1(2+3) - 4(2+3) + 1(4-0)$

$= 6 - 20 + 4 = -10.$

$$\Delta_3 = \begin{vmatrix} 1 & -1 & 4 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix}$$

$= 2 + 4 + 4$

$= 10$

$x = \frac{\Delta_1}{\Delta} = \frac{20}{10} = 2$

$y = \frac{\Delta_2}{\Delta} = \frac{-10}{10} = -1$

$z = \frac{\Delta_3}{\Delta} = \frac{10}{10} = 1$

(ii) $x + 2y - 3z = 4$

$2x + 4y - 5z = 12$

$3x - y + z = 3$

Sol $\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 4 & -5 \\ 3 & -1 & 1 \end{vmatrix}$

$= (4-5) - 2(2+15) - 3(-2-12)$

$= -1 - 34 + 42$

$= 7.$

$$\Delta_1 = \begin{vmatrix} 4 & 2 & -3 \\ 12 & 4 & -5 \\ 3 & -1 & 2 \end{vmatrix}$$

$= 4(4-5) - 2(12+15) - 3(-12-12)$

$= -4 - 54 + 72$

$= 14$

$$\Delta_2 = \begin{vmatrix} 1 & 4 & -3 \\ 2 & 12 & -5 \\ 3 & 3 & 1 \end{vmatrix}$$

$= (12+15) - 4(2+15) - 3(6-36)$

$= 27 - 68 + 90$

$= 49.$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 4 \\ 2 & 4 & 12 \\ 3 & -1 & 3 \end{vmatrix}$$

$= (12+12) - 2(6-36) + 4(-2-12)$

$= 24 + 60 - 56$

$= 28$

$x = \frac{\Delta_1}{\Delta} = \frac{14}{7} = 2$

$y = \frac{\Delta_2}{\Delta} = \frac{49}{7} = 7$

$z = \frac{\Delta_3}{\Delta} = \frac{28}{7} = 4.$

(i) **Trivial Solution:-**

→ When $x=0$, $y=0$ and $z=0$, then the solution is called trivial solution.

i.e $\Delta \neq 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$

(ii) **Non-Trivial Solution:-**

→ When atleast one of the variable is not equal to zero.

i.e $\Delta \neq 0$, atleast one of the $\Delta_1, \Delta_2, \Delta_3$ is not equal to zero.

(iii) **Unique Solution:-**

→ If $\Delta \neq 0$ then the solution is unique and consistant.

(iv) **Infinite/many Solution:-**

→ ~~In consistant~~ $\Delta = 0$.

→ If $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$ then the solution is infinitely many solution and consistant.

(v) **In Consistant:-**

→ In Consistant $\Delta = 0$, with atleast one of the $\Delta_1, \Delta_2, \Delta_3$ is not equal to zero.

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Exercise-5(b)

Solve by Cramer's Rule:-

(i) ~~$x-y+z=4$~~
~~$2x+y-3z=0$~~
~~$x+y+z=2$~~

(ii) $2x-y+z=4$
$x+3y+2z=12$
$3x+2y+3z=16$

$$\Delta = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix}$$

$= 2(9-4)+1(3-6)+1(2-9)$

$= (2\times 5)+(1\times -3)+1(-7)$

$= 10-3-7$

$= 0$

$$\Delta_1 = \begin{vmatrix} 4 & -1 & 1 \\ 12 & 3 & 2 \\ 16 & 2 & 3 \end{vmatrix}$$

$= 4(9-4)+1(36-32)+1(24-48)$

$= (4\times 5)+1(4)+1(-24)$

$= 20+4-24$

$= 24-24$

$= 0$

$$\Delta_2 = \begin{vmatrix} 2 & 4 & 1 \\ 1 & 12 & 2 \\ 3 & 16 & 3 \end{vmatrix}$$

$\Rightarrow \Delta_2 = 2(36-32)-4(3-6)+1(16-36)$

~~$=(2\times 4)-(4\times -3)+$~~

$= 2\times 4 - 4\cdot -3 + 1\cdot -20$

$= 8+12-20$

$= 20-20$

$= 0$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 4 \\ 1 & 3 & 12 \\ 3 & 2 & 16 \end{vmatrix}$$

$= 2(48-24)+1(16-36)+4(2-9)$

$= (2\times 24)+(1\cdot -20)+(4\cdot -7)$

$= 48-20-28$

$= 48-48$

$= 0$

∴ Since $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$, then the solution is infinitely many solution and consistant. because $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

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(iv) $x+y+z=4$
$2x+5y-2z=3$
$x+7y-7z=5$

Here $a_1=1$, $a_2=2$, $a_3=3$; $b_1=1$, $b_2=5$, $b_3=7$; $c_1=1$, $c_2=-2$, $c_3=-7$

$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & -2 \\ 1 & 7 & -7 \end{vmatrix}$$

$\Rightarrow \Delta = 1(-35+14) - 1(-14+2) + 1(14-5)$

$\Rightarrow \Delta = (1 \times -21) - 1(-12) + (1 \times 9)$

$= -21 + 12 + 9$

$\boxed{\Delta = 0}$

$$\Delta_1 = \begin{vmatrix} 4 & 1 & 1 \\ 3 & 5 & -2 \\ 5 & 7 & -7 \end{vmatrix}$$

$= 4(-35+14) - 1(-21+10) + 1(21-25)$

$= (4 \times -21) - 1(-11) + 1(-4)$

$= -84 + 11 - 4$

$= 11 - 88$

$= -77$

$$\Delta_2 = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 3 & -2 \\ 1 & 5 & -7 \end{vmatrix}$$

$= 1(-21+10) - 4(-14+2) + 1(10-3)$

$= (1 \times -11) - 4 \cdot (-12) + (1 \cdot 7)$

$= -11 + 48 + 7$

$= 48 - 4$

$= 44$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 5 & 3 \\ 1 & 7 & 5 \end{vmatrix}$$

$= 1(25-21) - 1(10-3) + 4(14-5)$

$= 1 \times 4 - 1 \cdot 7 + 4 \cdot 9$

$= 4 - 7 + 36$

$= 40 - 7$

$= 33$

∴ The Solution is a non consistant Solution because $\Delta = 0$ & atleast Δ_1 / Δ_2 & $\Delta_3 \neq 0$.

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Q) $x+y+z=4$
$2x-3y+3z=1$
$3x+2y-z=1.$

Here
$a_1=1 \quad b_1=1 \quad c_1=1 \quad d_1=4$
$a_2=2 \quad b_2=-1 \quad c_2=3 \quad d_2=1$
$a_3=3 \quad b_3=2 \quad c_3=-1 \quad d_3=1$

Now
$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{vmatrix}$$

$\Rightarrow \Delta = 1(1-6) - 1(-2-9) + 1(4+3)$

$= (1 \cdot -5) - 1 \cdot (-11) + 1 \cdot 7$

$= -5 + 11 + 7$

$= 11 + 2$

$\Rightarrow \Delta = 13$

$$\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 4 & 1 & 1 \\ 1 & -1 & 3 \\ 1 & 2 & -1 \end{vmatrix}$$

$= 4(1-6) - 1(-1-3) + 1(2+1)$

$= (4 \times -5) - 1(-4) + 1 \cdot 3$

$= -20 + 4 + 3$

$= -13$

$$\Rightarrow \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & -1 \end{vmatrix}$$

$\Rightarrow \Delta_2 =$

$= 1(-1-3) - 4(-2-9) + 1(2-3)$

$= 1(-4) - 4 \cdot (-11) + 1 \cdot (-1)$

$= -4 + 44 - 1$

$= 44 - 5$

$= 39$

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 4 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$= 1(-1-2) - 1(2-3) + 4(4+3)$

$= 1 \cdot (-3) - 1(-1) + 4 \cdot 7$

$= -3 + 1 + 28$

$= 1 + 25 = 26$

Now x.

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Now $\Rightarrow x = \frac{\Delta_1}{\Delta} = \frac{-13}{13} = -1$

$y = \frac{\Delta_2}{\Delta} = \frac{39}{13} = 3$

$z = \frac{\Delta_3}{\Delta} = \frac{26}{13} = 2.$

Since $\Delta \neq 0$, then the solution is unique and consistent.

$x + y - z = 6$

$2x - 3y + z = 1$

$2x - 4y + 2z = 1$

Here $a_1 = 1$, $b_1 = 1$, $c_1 = -1$, $d_1 = 6$

$a_2 = 2$, $b_2 = -3$, $c_2 = 1$, $d_2 = 1$

$a_3 = 2$, $b_3 = -4$, $c_3 = 2$, $d_3 = 1$

Now $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & -1 \\ 2 & -3 & 1 \\ 2 & -4 & 2 \end{vmatrix}$

$= 1(-6+4) - 1(4-2) - 1(-8+6)$

$= (1 \times -2) - 1(2) - 1(-2)$

$= -2 - 2 + 2$

$= -2.$

Now $\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

$= \begin{vmatrix} 6 & 1 & -1 \\ 1 & -3 & 1 \\ 1 & -4 & 2 \end{vmatrix}$

$= 6(-6+4) - 1(2-1) + 1(-4+3)$

$= 6(-2) - 1(1) + 1 \cdot (-1)$

$= -12 - 1 - 1$

$= -60 + 1 + 7$

$= -60 + 8 - 12 + 6$

$= -54.$

$= 6(-6+4) - 1(2-1) - 1(-4+3)$

$= 6(-2) - 1 \cdot 1 - 1(-1)$

$= -12 - 1 + 1$

$= -12$

$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & 6 & -1 \\ 2 & 1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

$= 1(2-1) - 6(4-2) - 1(2-2)$

$= 1 - 12 - 0$

$= -11$

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$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 6 \\ 3 & -3 & 1 \\ 2 & -4 & 1 \end{vmatrix}$$

$= 1(-3+4) - 1(3-2) + 6(-8+6)$

$= 1 - 0 - 12$

$= -11$

Now $x = \frac{\Delta_1}{\Delta} = \frac{-12}{-2} = 6$

$y = \frac{\Delta_2}{\Delta} = \frac{-11}{-2} = \frac{11}{2} = 5.5$

$z = \frac{\Delta_3}{\Delta} = \frac{-11}{-2} = \frac{11}{2} = 5.5$

Since $\Delta \neq 0$, the Solution is unique and Consistant.

(vii) $x - 2y = 3$

$3x + 4y - z = -2$

$5x - 3z = -1$

Here

$a_1 = 1$, $b_1 = -2$, $c_1 = 0$, $d_1 = 3$

$a_2 = 3$, $b_2 = 4$, $c_2 = -1$, $d_2 = -2$

$a_3 = 5$, $b_3 = 0$, $c_3 = -3$, $d_3 = -1$

Now $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 0 \\ 3 & 4 & -1 \\ 5 & 0 & -3 \end{vmatrix}$

$= 1(-12-0) + 2(-9+5) + 0$

$= -12 - 8$

$= -20$

$\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 3 & -2 & 0 \\ -2 & 4 & -1 \\ -1 & 0 & -3 \end{vmatrix}$

$= 3(-12-0) + 2(6-1)$

$= -36 + (2 \times 5)$

$= -36 + 10$

$= -26$

$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 0 \\ 3 & -2 & -1 \\ 5 & -1 & -3 \end{vmatrix}$

$= 1(6-1) - 3(-9+5) + 0$

$= 5 + 12$

$= 17$

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$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 3 \\ 3 & 4 & -2 \\ 5 & 0 & 7 \end{vmatrix}$$

$$= 1(-4-0) + 2(-3+10) + 3(0-20)$$

$$= -4 + 2 \cdot 7 + 3 \cdot -20$$

$$= -4 + 14 - 60$$

$$= 10 - 60$$

$$= -50$$

Now $x = \frac{\Delta_1}{\Delta} = \frac{-26}{-20} = \frac{13}{10} = 1.3$

$y = \frac{\Delta_2}{\Delta} = \frac{17}{-20} = -\frac{17}{20} = -0.85$

$z = \frac{\Delta_3}{\Delta} = \frac{-50}{-20} = \frac{5}{2} = 2.5$

Since $\Delta \neq 0$, the solution is unique and consistant.

(Ans)

$$2x + 3y + z = 11$$
$$x + y + z = 6$$
$$5x - y + 10z = 34$$

Here $a_1 = 2$, $b_1 = 3$, $c_1 = 1$, $d_1 = 11$

$a_2 = 1$, $b_2 = 1$, $c_2 = 1$, $d_2 = 6$

$a_3 = 5$, $b_3 = -1$, $c_3 = 10$, $d_3 = 34$

Now $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 1 & 1 \\ 5 & -1 & 10 \end{vmatrix}$

$$= 2(10+1) - 3(10-5) + 1(-1-5)$$

$$= (2 \times 11) - 3 \cdot (5) + 1 \cdot (-6)$$

$$= 22 - 15 - 6$$

$$= 7 - 6$$

$$= 1$$

$$\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 11 & 3 & 1 \\ 6 & 1 & 1 \\ 34 & -1 & 10 \end{vmatrix}$$

$$= 11(10+1) - 3(60-34) + 1(-6-34)$$

$$= (11 \times 11) - 3 \cdot (26) + (1 \cdot -40)$$

$$= 121 - 78 - 40$$

$$= 121 - 118$$

$$= 3$$

$$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} = \begin{vmatrix} 2 & 11 & 1 \\ 1 & 6 & 1 \\ 5 & 34 & 10 \end{vmatrix}$$

$$= 2(60-34) - 11(10-5) + 1(34-30)$$

$$= (2 \times 26) - 11 \cdot (5) + 1 \cdot 4$$

$$= 52 - 55 + 4$$

$$= 56 - 55$$

$$= 1$$

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$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 11 \\ 1 & 1 & 6 \\ 5 & -1 & 34 \end{vmatrix}$$

$= 2(34+6) - 3(34-30) + 11(-1+5)$

$= (2 \times 40) - (3 \cdot 4) + (11 \cdot 4)(-6)$

$= 80 - 12 + 44 - 66$

$= 80 - 78$

$= 2$

Now $x = \frac{\Delta_1}{\Delta} = \frac{2}{1} = 2$

$y = \frac{\Delta_2}{\Delta} = \frac{1}{1} = 1$

$z = \frac{\Delta_3}{\Delta} = \frac{2}{1} = 2$

→ Since $\Delta \neq 0$ the solution is consistent and consistent.

(viii) $x + 2y + 3z = 14$

$2x - 3y + 5z = 15$

$2y + 4z - 3x = 13$

$-3x + 2y + 4z = 13$

Here $a_1 = 1$, $b_1 = 2$, $c_1 = 3$, $d_1 = 14$

$a_2 = 2$, $b_2 = -1$, $c_2 = 5$, $d_2 = 15$

$a_3 = -3$, $b_3 = 2$, $c_3 = 4$, $d_3 = 13$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \\ -3 & 2 & 4 \end{vmatrix}$$

$= 1(-4-10) - 2(8+15) + 3(4-3)$

$= -14 - 46 + 3$

$= -60 + 3$

$= -57$

$$\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 14 & 2 & 3 \\ 15 & -1 & 5 \\ 13 & 2 & 4 \end{vmatrix}$$

$= 14(-4-10) - 2(60-65) + 3(30+13)$

$= 14 \times (-14) - (2 \times 5) + 3(43)$

$= -196 + 10 + 129$

$= -196 + 139$

$= -57$

$$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & 14 & 3 \\ 2 & 15 & 5 \\ -3 & 13 & 4 \end{vmatrix}$$

$= 1(60-65) - 14(8+15) + 3(26+45)$

$= -5 - (14 \cdot 23) + 3(71)$

$= -5 - 322 + 213$

$= 213 - 327$

$= -114$

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$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 & 14 \\ 2 & -1 & 15 \\ -3 & 2 & 13 \end{vmatrix}$

$= 1(-13-30) - 2(26+45) + 14(4-3)$

$= 1(-43) - 2(71) + (14 \times 1)$

$= -43 - 142 + 14$

$= -185 + 14$

$= -171$

$x = \frac{\Delta_1}{\Delta} = \frac{+57}{+57} = 1$

$y = \frac{\Delta_2}{\Delta} = \frac{+114}{+57} = 2$

$z = \frac{\Delta_3}{\Delta} = \frac{-171}{+57} = -3$

$\therefore$ Since $\Delta \neq 0$ the solution is unique & consistant.

★ Solution of system of linear homogeneous equations:-

Suppose $a_1x + b_1y + c_1z = 0$

$a_2x + b_2y + c_2z = 0$

$a_3x + b_3y + c_3z = 0$

$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Here $\Delta_1 = \Delta_2 = \Delta_3 = 0$.

Case-I

If $\Delta \neq 0$

$\Rightarrow x = 0, y = 0, z = 0$

$x = y = z = 0$ (Trivial solution)

$x = \frac{\Delta_1}{\Delta} = \frac{0}{\Delta} = 0$.

$y = \frac{\Delta_2}{\Delta} = \frac{0}{\Delta} = 0$

$z = \frac{\Delta_3}{\Delta} = \frac{0}{\Delta} = 0$

Case-II

If $\Delta = 0$

$\Rightarrow \Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

There is infinitely many solution.

If $\Delta = 0$ then we take one of the variable is taken as constant.

Suppose $z = k$. (say)

$\Rightarrow a_1x + b_1y + c_1k = 0$

$a_2x + b_2y + c_2k = 0$

$a_3x + b_3y + c_3k = 0$

Solving these three equation we get

$x = \alpha$ & $y = \beta$

So the solution is $x = \alpha, y = \beta, z = k$,

where $k \in \mathbb{R}$.

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Note:-

* If no. of eqn < no. of variable then either solution does not exist or infinitely many solution.

eg: $2x+3y+4z=5$
$2x+5y+7z=$

e.g. * $x-3y=0$

$3y=x$

$\Rightarrow y=\frac{x}{3}$.

$\Rightarrow y=\frac{1}{3}\cdot x$ (for this point is pass through the origin, because $y=mx$ where $m=\frac{1}{3}$) $\Rightarrow y=mx$

$\Rightarrow y=\frac{1}{3}(x=1)$

$y=\frac{2}{3}(x=2)$

Here solutions are (0,0), $(1, \frac{1}{3})$, $(2, \frac{2}{3})$, $(3,1)$, $(4, \frac{4}{3})$, $(5, \frac{5}{3})$, $(6,2)$

no. of eqn. < no. of variable

$(1<2)$

Then the solution is infinite.

Exercise-5 (b)

1. Write the no. of solutions of the following system of equations.

(i) $x-2y=0$
$x=2y$.
equation is infinity.

(ii) $x-y=0$ & $2x-2y=1$

$x-y=0$ — (I)
$2x-2y=1$ — (II)

eqn (I) × 2 = $2x-2y=0$
eqn (II) $2x-2y=1$

Here $a_1=1$ $b_1=-1$ $c_1=0$
$a_2=2$ $b_2=-2$ $c_2=0$

$\Delta = \begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix}$

$= -2+2$

$\Rightarrow \Delta = 0$

$\Rightarrow \Delta_1 = \begin{vmatrix} 0 & -1 \\ 0 & 2 \end{vmatrix}$

$= 2\cdot 0-(-1)$

$\Rightarrow \Delta_1 = 1$

$\Delta_2 = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$

$\Rightarrow 1-0$

$\Rightarrow \Delta_2 = 1$.

Now $x=\frac{\Delta_1}{\Delta}=\frac{1}{0}=\infty$

$y=\frac{\Delta_2}{\Delta}=\frac{1}{0}=\infty$

∴ since $\Delta=0$ so the equation is inconsistant.

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(iii) $2x+y=2$.

$-x-\frac{1}{2}y=3$

here $a_1=2$ $b_1=1$ $d_1=2$

$a_2=-1$ $b_2=-1/2$ $d_2=3$.

Now $\Delta=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

$=\begin{vmatrix} 2 & 1 \\ -1 & -1/2 \end{vmatrix}$

$=-2/2+1$

$=-1+1$

$\Rightarrow \Delta=0$

$\Delta_1=\begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix}$

$=\begin{vmatrix} 2 & 1 \\ 3 & -1/2 \end{vmatrix}$

$=2-1/2-3$

$=-4$.

$\Delta_2=\begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix}$

$=\begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix}$

$\Rightarrow \Delta_2=6+2$

$\Rightarrow \Delta_2=8$

$x=\frac{\Delta_1}{\Delta}=\frac{-4}{0}=\infty$

$y=\frac{\Delta_2}{\Delta}=\frac{8}{0}=\infty$

∴ Since $\Delta=0$, & the ~~equation~~ solution is inconsistent

(iv) ~~$3x+2y+1=0$~~

$3x+2y=1$

$x+5y=6$.

here $a_1=3$ $b_1=2$ $d_1=1$

$a_2=1$ $b_2=5$ $d_2=6$

$\Delta=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

$=\begin{vmatrix} 3 & 2 \\ 1 & 5 \end{vmatrix}$

$=15-2$

$\Rightarrow \Delta=13$.

$\Delta_1=\begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix}$

$\Rightarrow \Delta_1=\begin{vmatrix} 1 & 2 \\ 6 & 5 \end{vmatrix}$

$=5-12$

$\Rightarrow \Delta_1=-6$

$\Delta_2=\begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix}$

$=\begin{vmatrix} 3 & 1 \\ 1 & 6 \end{vmatrix}$

$\Rightarrow 18-1$

$\Rightarrow \Delta_2=17$

Now $x=\frac{\Delta_1}{\Delta}=\frac{-6}{13}$

$y=\frac{\Delta_2}{\Delta}=\frac{17}{13}$

Since $\Delta\neq 0$ then the equation is unique solution.

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(v) $2x+3y+1=0 \Rightarrow 2x+3y=-1$

$x-3y-4=0 \Rightarrow x-3y=4$

$a_1=2 \quad b_1=3 \quad c_1=-1 \quad d_1=-1$

$a_2=1 \quad b_2=-3 \quad d_2=4$

Now $\Delta = \begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix}$

$= -6-3$

$= -9$

$\Delta_1 = \begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix}$

$= \begin{vmatrix} -1 & 3 \\ 4 & -3 \end{vmatrix}$

$= 3-12$

$= -9$

$\Delta_2 = \begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix}$

$= \begin{vmatrix} 2 & -1 \\ 1 & 4 \end{vmatrix}$

$= 8+1$

$= 9$

Now $x = \frac{\Delta_1}{\Delta} = \frac{-9}{-9} = +1$

$y = \frac{\Delta_2}{\Delta} = \frac{9}{-9} = -1$

∴ Since $\Delta \neq 0$ they the solution is unique

(vi) $x+y+z=1$

$x+y+z=2$

$2x+3y+z=0$.

here $a_1=1 \quad b_1=1 \quad c_1=1 \quad d_1=1$

$a_2=1 \quad b_2=1 \quad c_2=1 \quad d_2=2$

$a_3=2 \quad b_3=3 \quad c_3=1 \quad d_3=0$

now $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix}$

$= 1(1-3)-1(1-2)+1(3-2)$

$= 0-2+1+1$

$= 0$.

$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 3 & 1 \end{vmatrix} = 1(1-3)-1(2-0)+1(6-0)$

$= 1-2-2+6$

$\Rightarrow \Delta_1 = 2$.

$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 0 & 1 \end{vmatrix} = 1(2-0)-1(1-2)+1(0-4)$

$= 2+1-4$

$\Rightarrow \Delta_2 = -1$.

$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 0 \end{vmatrix} = 1(0-6)-1(0-4)+1(3-2)$

$= -6+4+1$

$= -1$.

Now $x = \frac{\Delta_1}{\Delta} = \frac{2}{0} = \infty$

$y = \frac{\Delta_2}{\Delta} = \frac{-1}{0} = \infty$

$z = \frac{\Delta_3}{\Delta} = \frac{-1}{0} = \infty$.

∴ Since $\Delta \neq 0$ then the solution is inconsistant.

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(vii) $x+4y-z=0$
$3x-4y-z=0$
$x-3y+z=0$.

Here $a_1=1$ $b_1=4$ $c_1=-1$ $d_1=0$
$a_2=3$ $b_2=-4$ $c_2=-1$ $d_2=0$
$a_3=1$ $b_3=-3$ $c_3=1$ $d_3=0$.

Now $\Delta=\begin{vmatrix}1&4&-1\\3&-4&-1\\1&-3&1\end{vmatrix}$

$\Rightarrow \Delta = 1(-4-3)-4(3+1)-1(-9+4)$

$\Rightarrow \Delta = -7-16+5$

$\Rightarrow \Delta = -18$

$\Delta_1=\begin{vmatrix}0&4&-1\\0&-4&-1\\0&-3&1\end{vmatrix}=0$

$\Delta_2=\begin{vmatrix}1&0&-1\\3&0&-1\\1&0&1\end{vmatrix}=0$

$\Delta_3=\begin{vmatrix}1&4&0\\3&-4&0\\1&-3&0\end{vmatrix}=0$

$x=\frac{\Delta_1}{\Delta}=\frac{0}{-18}=0$

$y=\frac{\Delta_2}{\Delta}=\frac{0}{-18}=0$

$z=\frac{\Delta_3}{\Delta}=\frac{0}{-18}=0$

Since $\Delta\neq 0$ & $\Delta_1=\Delta_2=\Delta_3=0$
then the solution is trivial solution

(viii) $x+y-z=0$
$3x-y-z=0$
$x-3y+z=0$.

$\Delta=\begin{vmatrix}1&1&-1\\3&-1&-1\\1&-3&1\end{vmatrix}$

$\Rightarrow 1(-1-3)-1(3+1)-1(-9+1)$

$\Rightarrow -4-4+8$

$\Rightarrow 0$

$\Delta_1=\begin{vmatrix}0&1&-1\\0&-1&-1\\0&-3&1\end{vmatrix}=0$

$\Delta_2=\begin{vmatrix}1&0&-1\\3&0&-1\\1&0&1\end{vmatrix}=0$

$\Delta_3=\begin{vmatrix}1&1&0\\3&-1&0\\1&-3&0\end{vmatrix}=0$.

Since $\Delta=0$ $\Delta_1=\Delta_2=0$ then the solution is infinitely many solution.

(ix) $a_1x+b_1y+c_1z=0$
$a_2x+b_2y+c_2z=0$
$a_3x+b_3y+c_3z=0$
given that $\Delta=\begin{vmatrix}a_1&b_1&c_1\\a_2&b_2&c_2\\a_3&b_3&c_3\end{vmatrix}=0$

then $\Delta_1=\begin{vmatrix}0&b_1&c_1\\0&b_2&c_2\\0&b_3&c_3\end{vmatrix}=0$

$\Delta_2=\begin{vmatrix}a_1&0&c_1\\a_2&0&c_2\\a_3&0&c_3\end{vmatrix}=0$

$\Delta_3=\begin{vmatrix}a_1&b_1&0\\a_2&b_2&0\\a_3&b_3&0\end{vmatrix}=0$.

Since $\Delta=\Delta_1=\Delta_2=\Delta_3=0$ then, the solution is infinitely many solution.

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Show that the following equations is inconsistent.

(a-b)x + (b-c)y + (c-a)z = 0

(b-c)x + (c-a)y + (a-b)z = 0

(c-a)x + (a-b)y + (b-c)z = 0

$$\Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$= 0$

$$\Delta_1 = \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 1 & a-b & b-c \end{vmatrix}$$

$= 1\,(b-c)(a-b) - (c-a)(c-a)$

$= ab - b^2 - (b-c)(c-a)$

$= ab - b^2 - ca + bc - \{c^2 - 2ac + a^2\}$

$= ab - b^2 + bc - ca - c^2 + 2ac - a^2 \neq 0$

$= a^2 + b^2 + c^2 - ab - bc - ca \neq 0$

$$\Delta_2 = \begin{vmatrix} a-b & 0 & c-a \\ b-c & 0 & a-b \\ c-a & 1 & b-c \end{vmatrix}$$

$= 1\,(a-b)^2 - (b-c)(c-a) \neq 0.$

$$\Delta_3 = \begin{vmatrix} a-b & b-c & 0 \\ b-c & c-a & 0 \\ c-a & a-b & 1 \end{vmatrix}$$

$= 1\,(c-a)(a-b) - (b-c)^2 \neq 0$

Since $\Delta = 0$, then the Solution is inconsistent.

(1) The system of equations

$x + 2y + 3z = 4$

$2x + 3y + 4z = 5$

$3x + 4y + 5z = 6.$

$$\Delta_0 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 1(15-16) - 2(10-12) + 3(8-9)$$

$= -1 + 4 - 3$

$\Rightarrow \Delta = 0.$

$$\Delta_1 = \begin{vmatrix} 4 & 2 & 3 \\ 5 & 3 & 4 \\ 6 & 4 & 5 \end{vmatrix} = 4(15-16) - 2(25-24) + 3(20-18)$$

$= -4 - 2 + 6 = 0.$

$$\Delta_2 = \begin{vmatrix} 1 & 4 & 3 \\ 2 & 5 & 4 \\ 3 & 6 & 5 \end{vmatrix} = 1(25-24) - 4(10-12) + 3(12-15)$

$= 1 + 8 - 9$

$= 0.$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 4 \\ 2 & 3 & 5 \\ 3 & 4 & 6 \end{vmatrix} = 1(18-20) - 2(12-15) + 4(8-9)$

$= -2 + 6 - 4 = 0$

Since $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

then the Solution is infinitely many Solution

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10. (ii) If the system of equation.

$2x+5y+8z=0$
$x+y+7z=6$
$6x+9y-\lambda z=0$

has the non-trivial solution, then what is the value of λ

$$A=\begin{vmatrix} 2 & 5 & 8 \\ 1 & 1 & 7 \\ 6 & 9 & -\lambda \end{vmatrix}$$

$=2(-\lambda-63)-5(-\lambda-42)+8(9-6)$

$\Rightarrow -2\lambda-126+5\lambda+210+24$

$\Rightarrow -3\lambda-126+210+24$

$\Rightarrow -3\lambda+108=0$

$\Rightarrow -3\lambda=-108$

$\Rightarrow \lambda=36$

$2(-\lambda-63)-5(-\lambda-42)+8(9-24)$

$=-8\lambda-126+5\lambda+210-120$

$\Rightarrow -3\lambda-246+210=0$

$\Rightarrow -3\lambda=246-210$

$\Rightarrow 3\lambda=210-246 \Rightarrow \lambda=\frac{-36}{3}=-12$ (Ans)

(iii) $x+y+z=2$
$2x+y-z=3$
$3x+2y+kz=4$.

$$\Delta=\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix}$$

$=1(k+2)-1(2k+3)+1(4-3)$

$=k+2-2k-3+1$

$\Rightarrow -k=0$

$\Rightarrow k=0$.

(iv) $x+y+z=6$
$x+2y+3z=10$
$x+2y+mz=n$.

$$\Delta=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & m \end{vmatrix}$$

$=1(2m-6)-1(m-3)+1(2-2)$

$\Rightarrow 2m-6-m+3=0$

$\Rightarrow m-3=0$

$\Rightarrow m=3$.

(v) $\Delta_1=\begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ n & 2 & 3 \end{vmatrix}=0$

$\Rightarrow \Delta_1=6(6-6)-1(30-3n)+1(20-2n)$

$\Rightarrow \Delta_1=-30+3n+20-2n$

$\Rightarrow n-10=0$

$\Rightarrow n=10$

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(i) (v)

$2x - y + z = 0$

$x - 2y + z = 0$

$\lambda x - y + 2z = 0$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & -2 & 1 \\ \lambda & -1 & 2 \end{vmatrix}$$

~~$= 2(-2+1) - 1(2-\lambda) + 1(2-\lambda)$~~

$= 2(-4+1) + 1(2-\lambda) + 1(-1+2\lambda)$

$= -6 + 2 - \lambda - 1 + 2\lambda = 0$

$\Rightarrow \lambda - 5 = 0$

$\Rightarrow \lambda = 5$

(vi) $a_1 x + b_1 y + c_1 z = 0$

$a_2 x + b_2 y + c_2 z = 0$

$a_3 x + b_3 y + c_3 z = 0$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \quad \text{(Given)}$$

$$\Delta_1 = \begin{vmatrix} 0 & b_1 & c_1 \\ 0 & b_2 & c_2 \\ 0 & b_3 & c_3 \end{vmatrix} = 0$$

$$\Delta_2 = \begin{vmatrix} a_1 & 0 & c_1 \\ a_2 & 0 & c_2 \\ a_3 & 0 & c_3 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} = 0$$

$\therefore$ Rank of $\Delta = \Delta_1 = \Delta_2 = \Delta_3$ infinity many solution (more than two

(ii)

$$\begin{vmatrix} y & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix}$$

$\Rightarrow (y-2)^2 = (3x-2) - x - 6.$

$\Rightarrow y = 3x - 2 - x - 6.$

$\Rightarrow y = 2x - 8$

$\Rightarrow x = 6.$

(iii)

$$\begin{vmatrix} x & y \\ 1 & 1 \end{vmatrix} = 2, \quad \begin{vmatrix} x & 3 \\ y & 2 \end{vmatrix} = 1$$

$x - y = 2 \quad (I)$

$2x - 3y = 1 \quad (II)$

eqn (I) $\times 2$: $2x - 2y = 4$

eqn (II): $2x - 3y = 1$

$(-) \quad (+) \quad (-)$

$y = 3$

Put the value of x & y in eqn (I)

$x - 3 = 2$

$\Rightarrow x = 5.$

(iv)

$$\begin{vmatrix} x+1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} = 4$$

$C_1 \to C_1 - C_2$

$C_2 \to C_2 - C_3$

$$\begin{vmatrix} x & 0 & 1 \\ 0 & 2 & -1 \\ -2 & 0 & 1 \end{vmatrix}$$

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$n(2) + 1(4) = 4$

$\Rightarrow 2n + 4 = 4$

$\Rightarrow n = 0$

(v) $\begin{vmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{vmatrix}$

$= 0$

$= h(gf) - g(hf)$

$= 0 \cdot fgh - fgh$

$= 0$

(vi) $\begin{vmatrix} \frac{1}{a} & 1 & bc \\ \frac{1}{b} & 1 & ca \\ \frac{1}{c} & 1 & ab \end{vmatrix} =$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$= \begin{vmatrix} \frac{1}{a} - \frac{1}{b} & 0 & bc - ca \\ \frac{1}{b} - \frac{1}{c} & 0 & ca - ab \\ \frac{1}{c} & 1 & ab \end{vmatrix}$

$= \begin{vmatrix} \frac{b-a}{ab} & 0 & c(b-a) \\ \frac{c-b}{bc} & 0 & a(c-b) \\ \frac{1}{c} & 1 & ab \end{vmatrix}$

$= (b-a)(c-b) \begin{vmatrix} \frac{1}{ab} & 0 & c \\ \frac{1}{bc} & 0 & a \\ \frac{1}{c} & 1 & ab \end{vmatrix}$

$= (b-a)(c-b) \left(\frac{1}{ab} \cdot (-a) + \frac{1}{bc} \right)$

$= (b-a)(c-b) \left(-\frac{1}{b} + \frac{1}{b} \right)$

$= (b-a)(c-b) \cdot 0$

$= 0$.

(vii) $\begin{vmatrix} 19 & 3 & 24 \\ 17 & 9 & 34 \\ 25 & 9 & 50 \end{vmatrix}$

taking common 2 in C_3

$= 2 \begin{vmatrix} 19 & 3 & 19 \\ 17 & 9 & 17 \\ 25 & 9 & 25 \end{vmatrix}$

$= 2 \times 0$

$= 0$

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(ii) $\begin{vmatrix} 16 & 19 & 13 \\ 15 & 18 & 12 \\ 14 & 17 & 11 \end{vmatrix}$

$= C_2 \to C_2 - C_1$

$C_2 \to C_2 - C_1$

$\begin{vmatrix} 3 & 19 & 13 \\ 3 & 18 & 12 \\ 3 & 17 & 11 \end{vmatrix}$

$R_1 \to R_1 - R_2 \quad \& R_2 \to R_2 - R_3$

$= \begin{vmatrix} 0 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 17 & 11 \end{vmatrix}$

$= 3 \begin{vmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 17 & 11 \end{vmatrix}$

$= 3 \{ 1(1-1) \}$

$= 3 \times (1 \times 0)$

$= 0.$

(iii) $\begin{vmatrix} 224 & 777 & 32 \\ 735 & 888 & 105 \\ 812 & 999 & 116 \end{vmatrix}$

$= \text{taking Common 111 in } C_2$

$= 111 \times \begin{vmatrix} 224 & 7 & 32 \\ 735 & 8 & 105 \\ 812 & 9 & 116 \end{vmatrix}$

$= 111 \times [224(928 - 945) - 7(8260 - 85260) + 32(6615 - 6496)$

$= 111 \times (224(-17) - 0 + 32 \times (-119)$

$= 111 \times$

(iv) $\begin{vmatrix} 11 & 1 \\ 23 & 4 \\ 34 & 6 \end{vmatrix}$

$C_1 \to C_1 - C_2$

$C_2 \to C_2 - C_3$

$\begin{vmatrix} 0 & 6 & 1 \\ -1 & -1 & 4 \\ -1 & -2 & 6 \end{vmatrix}$

$= 2 - 1 = 1$

(v) $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 8 & 16 & 20 \end{vmatrix}$

$= \text{taking 2 Common in } R_3$

$2 \begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 7 & 10 \end{vmatrix}$

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$R_1 \to R_1 + R_2$

$$= 2\begin{vmatrix} 4 & 7 & 10 \\ 3 & 5 & 7 \\ 4 & 7 & 10 \end{vmatrix}$$

$= 0$

(vi) $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

$= 1(225-256) - 4(100-144) + 9(64-81)$

$= -31 + 176 - 153$

$= -184 + 176$

$= -8.$

(vii) $\begin{vmatrix} 1 & 0 & -5853 \\ -7361 & 2 & 7361 \\ 1 & 0 & 4137 \end{vmatrix}$

$R_1 \to R_1 - R_3$

$$= \begin{vmatrix} 0 & 0 & -10000 \\ -7361 & 2 & 7361 \\ 1 & 0 & 4137 \end{vmatrix}$$

$= -10000(-2)$

$= 20000$

(viii) $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix}$

$= C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$

$$\begin{vmatrix} 46 & 21 & 219 \\ 42 & 27 & 198 \\ 38 & 17 & 181 \end{vmatrix}$$

$C_1 \to C_1 - 2C_2$

$\Rightarrow C_3 \to C_3 - 10C_2$

$$\begin{vmatrix} 4 & 21 & 9 \\ -12 & 27 & -72 \\ 4 & 17 & 11 \end{vmatrix}$$

$R_1 \to R_1 - R_2$, $R_3 \to R_2 + 3R_3$

$$\Rightarrow \begin{vmatrix} 0 & 4 & -2 \\ 0 & 78 & -3 \\ 9 & 17 & 11 \end{vmatrix}.$$

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(viii) $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix}$

$= C_1 \to C_1 - C_2 \quad C_2 \to C_2 - C_3$

$\begin{vmatrix} 25 & 21 & 219 \\ 15 & 27 & 198 \\ 21 & 17 & 181 \end{vmatrix}$

$=$ taking 3 common from R_2

$\begin{vmatrix} 25 & 21 & 219 \\ 5 & 9 & 66 \\ 21 & 17 & 181 \end{vmatrix}$

$= C_1 \to C_1 - C_2$

$\begin{vmatrix} 4 & 21 & 219 \\ -4 & 9 & 66 \\ 4 & 17 & 181 \end{vmatrix}$

$C_3 \to C_3 \cdot R_3 \to R_3 - R_2$ & $R_2 \to R_2 + R_3$

$\begin{vmatrix} 0 & 12 & 153 \\ 0 & 26 & 247 \\ 4 & 17 & 181 \end{vmatrix}$

$= 4 \begin{vmatrix} 12 & 153 \\ 26 & 247 \end{vmatrix}$

$= 4\{(12 \times 247) - (153 \times 26)\}$

$= 4\{2964 - 2964\}$

$= 0.$

(ix) $\begin{vmatrix} 0 & a^2 & b \\ b^2 & 0 & a^2 \\ a & b^2 & 0 \end{vmatrix}$

$= -a^2(+a^3) + b(b^4)$

$= -a^5 + b^5$

$= b^5 - a^5.$

(x) $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix}$

$= C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix} 0 & b-c & c-a \\ 0 & y-z & z-x \\ 0 & q-r & r-p \end{vmatrix}$

$= 0$

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(xi) $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

$\Rightarrow C_1 \to C_1 + C_2 + C_3$

$\begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$

$= 0$

Ans 3 $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

$C_1 \to C_1 - C_2, \; C_2 \to C_2 - C_3$

$\begin{vmatrix} 0 & 0 & 1 \\ -x & x & 1 \\ 0 & -y & 1+y \end{vmatrix}$

$= xy = 0$

$\Rightarrow x = 0, \; y = 0$

Ans 4 $\begin{vmatrix} 2x & 0 & 0 \\ 0 & 1 & 2 \\ -1 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & y \\ 0 & 8 & 5 \end{vmatrix}$

$\cdot 2x(0-4) = 1(15-12)$

$= -8x = 3$

$\Rightarrow x = -\frac{3}{8}$

Ans 5 $\begin{vmatrix} x+a & 0 & 0 \\ a & x+b & 0 \\ a & 0 & x+c \end{vmatrix}$

$= (x+a)(x+b)(x+c) = 0$

$\Rightarrow x = -a, \; x = -b, \; x = -c.$

Ans 6 $\begin{vmatrix} a+x & a-x & a-y \\ a-y & a+y & a-y \\ a-x & a-x & a+y \end{vmatrix} = 0$

$C_2 \to C_2 - C_3$

$\begin{vmatrix} a+x & 0 & a-y \\ a-y & 2y & a-y \\ a+x & -2x & a+y \end{vmatrix}$

Taking Common $2x$ in C_2

$2x \begin{vmatrix} a+y & 0 & a-y \\ a-y & 1 & a-y \\ a-x & -1 & a+y \end{vmatrix} = 0$

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taking common

$R_2 \to R_2 + R_3$

$$= 2y \begin{vmatrix} a+x & 0 & a-y \\ 2(a-y) & 0 & 2y \\ a-x & -1 & a+x \end{vmatrix}$$

$= C_1 \to C_1 - C_3$

$$= 2y \begin{vmatrix} 2y & 0 & a-y \\ -2y & 0 & 2y \\ -2x & -1 & a+x \end{vmatrix}$$

$$= 4xy \begin{vmatrix} y & 0 & a-y \\ -y & 0 & 2y \\ -x & -1 & a+x \end{vmatrix}$$

$\Rightarrow (-1)\, 4xy\, (2y^2 + ay - xy)$

$= x^2 - xa - 2y^2 = 0$

$= x(x+a) - 2a(x+a)$

$= (x+a)(x-2y)$

$= x = -a \;/\; x = 2a,$

12.

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & x \\ x+a+b+c & b & x+c \end{vmatrix}$$

$$= (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix}$$

$\Rightarrow R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$(x+a+b+c) \begin{vmatrix} 0 & -x & 0 \\ 0 & x & -x \\ 1 & b & x+c \end{vmatrix}$$

$= (x+a+b+c)(x^2)$

$\Rightarrow x = 0 \;/\; x = -(a+b+c)$

19.

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

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$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & a-b & c(b-a) \\ 0 & b-c & a(c-b) \\ 1 & c & ab \end{vmatrix} - \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

= taking $(a-b)(b-c)$ in R_1 & R_2

$$(a-b)(b-c)\begin{vmatrix} 0 & 1 & -c \\ 0 & 1 & -a \\ 1 & c & ab \end{vmatrix} - (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & -c \\ 0 & 1 & -a \\ 1 & c & ab \end{vmatrix} - \begin{vmatrix} 0 & 1 & b+c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a-b)(b-c)(-a+c) - (b+c-a-b)$$

$$= (a-b)(b-c)(-a+c-b-c+a+b)$$

$$= 0$$

Ans

$$\begin{vmatrix} a & a^2-bc & 1 \\ b & b^2-ac & 1 \\ c & c^2-ab & 1 \end{vmatrix}$$

$R_1 \to R_1 - R_2$; $R_2 \to R_2 - R_3$

$$\begin{vmatrix} a-b & a^2-bc-b^2+ac & 0 \\ b-c & b^2-ac-c^2+ab & 0 \\ c & c^2-ab & 1 \end{vmatrix}$$

$\to R_1 \to R_1 + R_2$

$$\begin{vmatrix} a-b & (a-b)(a+b)+c(a-b) & 0 \\ b-c & (b-c)(b+c)+a(b-c) & 0 \\ c & c^2-ab & 1 \end{vmatrix}$$

= taking Common $(a-b)(b-c)$ from R_1 & R_3

$$(a-b)(b-c)\begin{vmatrix} 1 & a+b+ac & 0 \\ 1 & ba+b+c & 0 \\ c & c^2-ab & 1 \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$= \begin{vmatrix} 0 & 0 & 0 \\ 1 & a-b+c & 0 \\ c & c^2-ab & 1 \end{vmatrix}$$

$= 0.$

Ans

$x+y+z=6$

$4x+\lambda y-7z=0$

$3x+2y-4z=-5$

This solution is consistant so $\Delta = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & 7 \\ 3 & 2 & -4 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 4-\lambda & 2\lambda & 7 \\ 1 & 6 & -4 \end{vmatrix} = 0 \to 24 - 6\lambda - 2\lambda = 0$$

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$\Rightarrow 24 - 8\lambda = 0$

$\Rightarrow 24 = 8\lambda$

$\Rightarrow \frac{24}{8} = \lambda$

$\Rightarrow 3 = \lambda \Rightarrow \lambda = 3$ (Ans)

24) i) $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix}$

= multiply by abc and dividing abc

$= \frac{abc}{abc} \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix}$

$= \frac{1}{abc} \begin{vmatrix} a^3+a & a^2b & a^2c \\ ab^2 & b^3+b & b^2c \\ ac^2 & bc^2 & c^3+c \end{vmatrix}$

$\Rightarrow \frac{abc}{abc} \begin{vmatrix} a^2+1 & b^2 & c^2 \\ b^2 & b^2+1 & c^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

= taking common $R_1 \to R_1+R_2+R_3$

$= \begin{vmatrix} a^2+1+b^2+c^2 & a^2+b^2+1+c^2 & a^2+b^2+c^2+1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

= taking common in $a^2+b^2+c^2+1$ in R_1

$= a^2+b^2+c^2+1 \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$1+a^2+b^2+c^2 \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & b^2 \\ 0 & -1 & c^2+1 \end{vmatrix}$

$= (1+a^2+b^2+c^2)\, 1(1)$

$= 1+a^2+b^2+c^2$ Ans

(iii) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$R_1 \to R_1+R_2+R_3$

$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$

= taking common $a+b+c$ in R_1

$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix}$

$= (a+b+c)(ba-b^2-ca+cb-c^2+ca+ab-a^2)$

$= (a+b+c)(ab-b^2-ca+cb-c^2+2ca-a^2)$

$= (a+b+c)\{-(a^2+b^2+c^2-ca-ab-cb)\}$

$= -(a^3+ab^2+ac^2-ca^2-a^2b-abc+ba^2+b^3+bc^2-abc-ab^2-cb^2+ca^2+cb^2+c^3-c^2a-abc-c^2b)$

$= 3abc - a^3-b^3-c^3$ (Ans)

(iv) $\begin{vmatrix} b^2-ab & bc & bc-ac \\ ab-a^2 & ac & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix}$

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$$= \begin{vmatrix} b(b-a) & (b-c) & c(b-a) \\ a(b-a) & (a-b) & b(b-a) \\ c(b-a) & (c-a) & a(b-a) \end{vmatrix}$$

$$= (b-a)^2 \begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$$

$C_2 \to C_2 + C_3$

$$= (b-a)^2 \begin{vmatrix} b & b & c \\ a & a & b \\ c & c & a \end{vmatrix} = 0 \quad \text{(Ans)}$$

(N)
$$\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$$

taking a, b, c in C_1, C_2 & C_3

$$= abc \begin{vmatrix} -a & a & a \\ b & -b & b \\ c & c & -c \end{vmatrix}$$

$$= a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$R_1 \to R_1 + R_2$ & $R_2 \to R_2 + R_3$

$$= a^2b^2c^2 \begin{vmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 4a^2b^2c^2$$

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MATRIX :-

→ A matrix is rectangular array of no. arranged in rows and columns if there are 'm' rows and 'n' columns.

So there are total no. of element = $m \times n$ $[mn]$.

If 'A' is a matrix. having m rows and n columns then order of the matrix 'A' is $[m \times n]$.

→ A matrix is symmetrically denoted by either $[\ ]$ or $(\ )$.

e.g $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = 2 \times 3 = 6$ $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}_{2 \times 3}$.

→ Plural form of matrix is [matrices.]

→ The elements of the matrix is denoted by a_{ij}

i.e $A = [a_{ij}]$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3 \times 4}$$

Types of Matrix :-

(a) Zero Matrix :-

→ If all the elements of matrix is zero, then that matrix is called zero matrix.

→ It is also called null matrix.

→ It is denoted by $O_{m \times n}$.

eg $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3 \times 4} = O_{m \times n}$

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(b) **Row Matrix:-**

→ A matrix contain only one row is called row matrix.

eg → $A = [1 \ 2 \ 3 \ 4]_{1\times 4}$.

(c) **Coloumn Matrix:-**

→ A matrix having one coloumn is called coloumn matrix.

eg $A = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}_{4\times 1}$

(d) **Transpose Matrix:-**

→ Transpose of a matrix is obtained by changing rows are into coloumns or (coloumns are change to rows).

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$.

→ $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

→ If $A_{m\times n}$ is a matrix then its transpose denoted by A^T.

→ Transpose of 'A' is denoted by A^T.

$A \rightarrow A^T$

$(A^T)^T = A$

e.g: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$(A^T)^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = A$.

* **Note:-**

→ Transpose of a row matrix is a coloumn matrix and transpose of coloumn matrix is a row matrix.

eg $A = [1 \ 2 \ 3]_{1\times 3}$ → row matrix

$A^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$ → coloumn matrix.

$B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}_{3\times 1}$ → coloumn matrix.

$B^T = [4 \ 5 \ 6]_{1\times 3}$ → row matrix.

(e) **Diagonal Matrix:-**

→ A square matrix of which the non-diagonal elements are all zero, is called diagonal matrix.

eg $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{vmatrix}$ → Main diagonal.

here $\begin{vmatrix} 1 & x & x \\ x & 3 & x \\ x & x & 4 \end{vmatrix}$

i.e (i) $a_{ij} = 0$ for $j \neq i$

(ii) $a_{ij} \neq 0$ for $i = j$

* 1, 3, 4 is the called main diagonal of the determinant.

(f) **Scalar Matrix:-**

→ A diagonal matrix is called scalar matrix if the elements of diagonal are equal.

e.g → $\begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix}$

or $\begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$

i.e (i) $a_{ij} = 0$ for $j \neq i$

(ii) $a_{ij} = a_{ij}$ for $i = j$

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(g) Unit Matrix:-

→ A scalar matrix is called as unit matrix if all the diagonal matrix are unity.

e.g $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(h) Equality Of Matrices:-

→ Two matrices of the same order are said to be equal if and only if the corresponding elements of the two are equal.

e.g $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times3} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}$

$A = B$

$\Rightarrow a = 1, \quad b = 2, \quad c = 3$

$d = 4 \quad e = 5, \quad f = 6.$

(i) Square Matrix:-

→ A matrix is said to be square matrix if no. of rows equal to no. of columns.

i.e $m = n$.

e.g $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}_{3\times3}$

$\begin{bmatrix} a & b & c & d \\ b & c & d & e \\ c & d & e & f \\ h & i & j & k \end{bmatrix}_{m\times m} = \begin{bmatrix} a & b & c & d \\ b & c & d & e \\ c & d & e & f \\ h & i & j & k \end{bmatrix}_{m^2}$

Operation On Matrices:-

Operation On Matrices:-

Summation of matrices:-

→ The sum of two matrices of the same order is the matrix of which the elements are the sum of the corresponding element of the two matrices.

e.g $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times3}$

$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}$

$A+B = \begin{bmatrix} a+1 & b+2 & c+3 \\ d+4 & e+5 & f+6 \end{bmatrix}_{2\times3}$

Substraction of Matrices:-

→ The substraction of two matrices of the same elements order is the matrix of which the elements are the subtract of the corresponding elements of the two matrices.

e.g $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$A-B = \begin{bmatrix} a-1 & b-2 & c-3 \\ d-4 & e-5 & f-6 \end{bmatrix}$.

Scalar Multiple of a Matrix:-

→ If 'A' is a matrix and α is a scalar then αA is a matrix which is obtained by multiplying every element of matrix 'A' by α.

e.g $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$\alpha A = \begin{bmatrix} \alpha & 2\alpha & 3\alpha \\ 4\alpha & 5\alpha & 6\alpha \end{bmatrix}$

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☆ Additive inverse of a matrix:-

→ If 'A' is a matrix then '-A' is additive inverse matrix of 'A'.

eg: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ & $-A = \begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \end{bmatrix}$

→ $A+(-A)$

$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \end{bmatrix}$

$= \begin{bmatrix} 1-1 & 2-2 & 3-3 \\ 4-4 & 5-5 & 6-6 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3}$

→ If 'A' is a $m \times n$ matrix, then '-A' is also the $m \times n$ matrix.

→ Order of the zero matrix is same as order of A matrix.

☆ Properties involving addition of matrices:-

(a) Commutative law:-

→ If 'A' and 'B' are two matrices of same order then

$$\boxed{A+B = B+A}$$

(b) Associative law:-

→ If 'A', 'B', and 'C' are three matrices of same order then $\boxed{A+(B+C) = (A+B)+C}$

(c) Existance of identity:-

→ If 'A' is a matrix of $m \times n$ order then there exist zero matrix of $m \times n$ order such that

$$\boxed{A+0 = 0+A = A}$$

(d) Additive inverse:-

→ If 'A' is a matrix of $m \times n$ order then they are exist a matrix '-A' such that $\boxed{A+(-A) = 0 \text{ matrix}}$

☆ Properties of Scalar Multiplication:-

→ Let 'A' and 'B' are two matrices of same order and α, β are scalars, then

(i) $(\alpha+\beta)A = \alpha A + \beta A$

(ii) $\alpha(A+B) = \alpha A + \alpha B$

(iii) $\alpha(\beta A) = (\alpha\beta)A = \beta(\alpha A)$

MATRIX Multiplication:-

→ If two matrices 'A' and 'B' are comfortable multiplication if and only if the no. of columns in A is equal to the no. of rows in 'B'.

→ The product of two matrices denoted by AB.

$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

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$$AB = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} a_1\cdot1 + b_1\cdot4 + c_1\cdot7 & a_1\cdot2 + b_1\cdot5 + c_1\cdot8 & a_1\cdot3 + b_1\cdot6 + c_1\cdot9 \\ a_2\cdot1 + b_2\cdot4 + c_2\cdot7 & a_2\cdot2 + b_2\cdot5 + c_2\cdot8 & a_2\cdot3 + b_2\cdot6 + c_2\cdot9 \\ a_3\cdot1 + b_3\cdot4 + c_3\cdot7 & a_3\cdot3 + b_3\cdot5 + c_3\cdot8 & a_3\cdot3 + b_3\cdot6 + c_3\cdot9 \end{bmatrix}$$

→ If def

→ The product of matrix AB is defined as whose elements in i^{th} row and j^{th} column is the algebric sum of the product of the elements in the i^{th} row of a matrix 'A' by the corresponding elements in the j^{th} column of B.

→ If 'A' is of order $m \times n$ and 'B' is of order $n \times p$. then $A_{m\times n} \cdot B_{n\times p} = C_{m\times p}$

e.g $A_{2\times3} \cdot B_{3\times5} = C_{2\times5}$

★ Note:-

(i) If A & B are two matrices, such that A+B and AB are both defined, then A and B must be Square matrices of same order. $[A]_{3\times3} + [B]_{3\times3}$

(ii) If both AB and BA are defined and 'A' matrix of order $m \times n$, then B is a matrix of order $n \times m$.

★ Non-Singular Matrix:-

→ 'A' is a square matrix, said to be non-singular matrix iff det.A $\neq 0$ (determinant of A $\neq 0$) or $|A| \neq 0$

e.g:- $A = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}$

$|A| = 4\cdot2 - 3\cdot5$

$\Rightarrow |A| = 8 - 15$

$\Rightarrow |A| = -7 \neq 0$

or $\Rightarrow$ det.A $= -7 \neq 0$

Hence A is non-singular matrix.

★ Singular Matrix:-

→ 'A' is a square matrix said to be singular matrix iff det.A $= 0$ (determinant of A $= 0$) or $|A| = 0$

e.g $A = \begin{bmatrix} 8 & 2 \\ 2 & 1/2 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 8 & 2 \\ 2 & 1/2 \end{vmatrix}$

$\Rightarrow |A| = 8\times\frac{1}{2} - 2\cdot2$

$\Rightarrow |A| = 4 - 4$

$\Rightarrow |A| = 0$

or $\Rightarrow$ det.A $= 0$

hence A is singular-matrix.

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Properties of Matrix Multiplication:-

(i) Matrix multiplication in general is non-commutative (not always true)
i.e $AB \neq BA$.

(ii) Matrix multiplication is associative.
If A, B and C are be 3 matrices of order $m \times n$, $n \times p$ and $p \times q$ respectively.
then $(AB) \cdot C = A(BC)$

eg:- $A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 6 \\ 7 & -5 \end{bmatrix}$

$(AB) \cdot C$

$(AB) \cdot AB = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 \cdot 1 + (3 \cdot -3) & 2 \cdot 2 + 3 \cdot 1 \\ 5 \cdot 1 + 6 \cdot (-3) & 5 \cdot 2 + 6 \cdot 1 \end{bmatrix}$

$= \begin{bmatrix} -7 & 7 \\ -13 & 16 \end{bmatrix}$

$(AB) \cdot C = \begin{bmatrix} -7 & 7 \\ -13 & 16 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 7 & -5 \end{bmatrix}$

$= \begin{bmatrix} -7 \cdot 4 + 7 \cdot 7 & (-7) \cdot 6 + 7 \cdot (-5) \\ -13 \cdot 4 + 16 \cdot 7 & -13 \cdot 6 + 16 \cdot (-5) \end{bmatrix}$

$= \begin{bmatrix} -28 + 49 & -42 - 35 \\ -52 + 112 & -78 - 80 \end{bmatrix}$

$(AB) \cdot C = \begin{bmatrix} 21 & -77 \\ 60 & -158 \end{bmatrix}$ ——— (1)

$A \cdot (BC)$.

Again $BC = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 6 \\ 7 & -5 \end{bmatrix}$

$= \begin{bmatrix} 1 \cdot 4 + 2 \cdot 7 & 1 \cdot 6 + 2 \cdot -5 \\ -3 \cdot 4 + 1 \cdot 7 & +3 \cdot 6 + 1 \cdot (-5) \end{bmatrix}$

$= \begin{bmatrix} 18 & -4 \\ -5 & -23 \end{bmatrix}$

Now $A(BC) = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 18 & -4 \\ -5 & -23 \end{bmatrix}$

$= \begin{bmatrix} 2 \cdot 18 + 3 \cdot (-5) & 2(-4) + 3(-23) \\ 5 \cdot 18 + 6 \cdot (-5) & 5 \cdot (-4) + 6 \cdot (-23) \end{bmatrix}$

$= \begin{bmatrix} 36 - 15 & -8 - 69 \\ 90 - 30 & -20 - 138 \end{bmatrix}$

$(AB)C = \begin{bmatrix} 21 & -77 \\ 60 & -158 \end{bmatrix}$ ——— (11)

eq(1) = eq(11).

∴ Since $A(BC) = (AB) \cdot C$ So the matrix multiplication is associative.

(iii) Distributive law:-

→ Matrix multiplication is distributed over double matrix addition i.e $A(B+C) = AB + AC$.

⇒ If 'A' is a square matrix then $A^m \cdot A^n = A^{m+n}$, where $m, n \in N$

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→ $(A^m)^n = A^{mn}$

→ In general $A^n = A \cdot A \cdot A \cdot \ldots \ldots \ldots n$ times.

→ If I be the unit matrix (Identity matrix) of order n, n then $AI = IA = A$

★ Cancellation Law for multiplication:-

→ The equality $AB = AC$ does not necessarily implies that $B = C$

Q.a

eg →

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ -1 & 4 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ -1 & 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1\cdot1+2\cdot1+0\cdot2 & 1\cdot2+2\cdot1+0\cdot2 & 1\cdot3+2\cdot(-1)+0\cdot2 \\ 1\cdot1+1\cdot1+0\cdot2 & 1\cdot2+1\cdot1+0\cdot2 & 1\cdot3+1\cdot(-1)+0\cdot2 \\ -1\cdot1+4\cdot1+0\cdot2 & -1\cdot2+4\cdot1+0\cdot2 & -1\cdot3+4\cdot(-1)+0\cdot2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2+2 & 3-2 \\ 1+1 & 2+1 & 3-1 \\ -1+4 & -2+4 & -3-4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & 1 \\ 2 & 3 & 2 \\ 3 & 2 & -7 \end{bmatrix}$$

Now

$$AC = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ -1 & 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\cdot1+2\cdot1+0\cdot1 & 1\cdot2+2\cdot1+0\cdot1 & 1\cdot3+2(-1)+0\cdot1 \\ 1\cdot1+1\cdot1+0\cdot1 & 1\cdot2+1\cdot1+0\cdot1 & 1\cdot3+1\cdot(-1)+0\cdot1 \\ -1\cdot1+4\cdot1+0\cdot1 & -1\cdot2+4\cdot1+0\cdot1 & -1\cdot3+4(-1)+0\cdot1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2+2 & 3-2 \\ 1+1 & 2+1 & 3-1 \\ -1+4 & -2+4 & -3-4 \end{bmatrix} \quad \begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

$$AC = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 3 & 2 \\ 3 & 2 & -7 \end{bmatrix}$$

~~Since $AB = AC$ Q, prove the~~

∴ Since $AB = AC$ but $B \neq C$.

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→ If AB is a null matrix (zero matrix), then it does not necessarily imply that either A or B should be null matrix.

e.g. $A = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2\cdot 1 + 2\cdot(-1) & 2\cdot 1 + 2\cdot(-1) \\ 3\cdot(-1) + 3\cdot 1 & 3\cdot(-1) + 3\cdot 1 \end{bmatrix}$

$= \begin{bmatrix} 2-2 & 2-2 \\ -3+3 & -3+3 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

★ Determinant of the product of two square matrices :-

→ If A and B are square matrices of order, then

★ $|AB| = |A| \cdot |B|$

i.e $\det(AB) = \det A \cdot \det B$.

e.g → $A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2\cdot 1 + 4\cdot 3 & 2\cdot 2 + 4\cdot 4 \\ 3\cdot 1 + 5\cdot 3 & 3\cdot 2 + 5\cdot 4 \end{bmatrix}$

$= \begin{bmatrix} 14 & 20 \\ 18 & 26 \end{bmatrix}$

$= 364 - 360 = 4$

$\det(AB) = \begin{vmatrix} 14 & 20 \\ 18 & 26 \end{vmatrix}$

$\Rightarrow |AB| = 364 - 360$

$\Rightarrow |AB| = 360 + 4$

$\det(A) = \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$

$\Rightarrow |A| = 10 - 12$

$\Rightarrow |A| = -2$.

$\det(B) = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

$|B| = 4 - 6$

$\Rightarrow |B| = -2$

now $|AB| = |A| \cdot |B|$

$\Rightarrow 4 = (-2)(-2)$ (proved)

$\Rightarrow$

Ques: ★ Find matrix B, if $B^2 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$

Soln

$B^2 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$

$= 16 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow B^2 = 16I$ { I : Identity matrix (unit matrix) }

$\Rightarrow B^2 = 16 I^2$ { $I = I^2 = I^3 = I^4$ }

$\Rightarrow B = \pm 4I$

$\Rightarrow B = +4I$ or $-4I$.

$\Rightarrow B = 4 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ or.

$\Rightarrow B = -4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$

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Ex:

* If $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ then show that $A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$

(X)

~~let P_n be a~~

let $P_n: \begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$ be a given statement.

Prove

let $P_n: \begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$

Step-1 let $n=1$

P_1 is true.

$P_{(1)}: \begin{bmatrix}1+2\cdot1 & -4\cdot1\\ 1 & 1-2\cdot1\end{bmatrix}$

$= P_{(1)}: \begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$

Step-2

Assume that P_K is true.

$P_K = \begin{bmatrix}1+2k & -4k\\ k & 1-2k\end{bmatrix}$

Step-3

We have to prove that P^{k+1} is true.

$P^{k+1} = \begin{bmatrix}1+2(k+1) & -4(k+1)\\ k+1 & 1-2(k+1)\end{bmatrix}$

$= \begin{bmatrix}1+2k+2 & -4k-4\\ k+1 & 1-2k-2\end{bmatrix}$

$P^{k+1} = \begin{bmatrix}2k+3 & -4k-4\\ k+1 & -2k-1\end{bmatrix}$ ——— (1)

* Now LHS

P^{k+1}.

$= P^k \cdot P$

$= \begin{bmatrix}1+2k & -4k\\ k & 1-2k\end{bmatrix}\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$

$= \begin{bmatrix}(1+2k)3 + (-4k)\cdot1 & (1+2k)(-4)\cdot + (-4)(-1)\cdot\\ k\cdot3 + (1-2k)\cdot1 & k(-4) + (1-2k)(-1)\cdot\end{bmatrix}$

$= \begin{bmatrix}3+6k-4k & -4-8k+4k\\ 3k+1-2k & -4k+1+2k\end{bmatrix}$

$= \begin{bmatrix}2k+3 & -4k-4\\ k+1 & -2k-1\end{bmatrix}$ = R.H.S (proved)

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If $D_j = \begin{vmatrix} j & a & n(n+1)/2 \\ j^2 & b & n(n+1)(2n+1)/6 \\ j^3 & c & n^2(n+1)^2/4 \end{vmatrix}$

then prove that $\sum_{j=1}^{n} D_j = 0$

Prove

$$\sum_{j=1}^{n} D_j = D_1 + D_2 + D_3 + \cdots + D_n$$

$$= \begin{vmatrix} 1 & a & n(n+1)/2 \\ 1^2 & b & n(n+1)(2n+1)/6 \\ 1^3 & c & n^2(n+1)^2/4 \end{vmatrix} + \begin{vmatrix} 2 & a & n(n+1)/2 \\ 2^2 & b & n(n+1)(2n+1)/6 \\ 2^3 & c & n^2(n+1)^2/4 \end{vmatrix} + \begin{vmatrix} 3 & a & n(n+1)/2 \\ 3^2 & b & n(n+1)(2n+1)/6 \\ 3^3 & c & n^2(n+1)^2/4 \end{vmatrix} + \cdots + \begin{vmatrix} n & a & n(n+1)/2 \\ n^2 & b & n(n+1)(2n+1)/6 \\ n^3 & c & n^2(n+1)^2/4 \end{vmatrix}$$

$$= \begin{vmatrix} 1+2+3+\cdots+n & a & n(n+1)/2 \\ 1^2+2^2+3^2+\cdots+n^2 & b & n(n+1)(2n+1)/6 \\ 1^3+2^3+3^3+\cdots+n^3 & c & n^2(n+1)^2/4 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{n(n+1)}{2} & a & \frac{n(n+1)}{2} \\ \frac{n(n+1)(2n+1)}{6} & b & \frac{n(n+1)(2n+1)}{6} \\ \left(\frac{n(n+1)}{2}\right)^2 & c & \frac{n^2(n+1)^2}{4} \end{vmatrix}$$

$= 0$ (proved) $(\because C_1 = C_3)$

If x, y, z are the $+ve$ and are the p^{th}, q^{th} and r^{th} terms of a G.P then,

Prove that $\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = 0$

Prove

According to question x, y, z are the p^{th}, q^{th} and r^{th} term of G.P respectively.

i.e $t_p = ar^{p-1} = x$

$t_q = ar^{q-1} = y$

$t_r = ar^{r-1} = z$

$$\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = 0$$

Replacing $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$\begin{vmatrix} \log x - \log y & p-q & 0 \\ \log y - \log z & q-r & 0 \\ \log z & r & 1 \end{vmatrix}$$

$$= 1 \cdot \begin{vmatrix} \log x - \log y & p-q \\ \log y - \log z & q-r \end{vmatrix}$$

$$= \begin{vmatrix} \log \frac{x}{y} & p-q \\ \log \frac{y}{z} & q-r \end{vmatrix}$$

On G.P the terms are $a, ar^2, ar^3, \ldots$

$t_1 = a$

$t_2 = ar$

$t_3 = ar^2$

$t_4 = ar^{4-1} = ar^3$

$t_5 = ar^{5-1} = ar^4$

$t_n = ar^{n-1}$ where r is common ratio.

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$$= \begin{vmatrix} \log\left(\frac{ar^{p-1}}{ar^{q-1}}\right) & p-q \\ \log\left(\frac{ar^{q-1}}{ar^{r-1}}\right) & q-r \end{vmatrix}$$

$$= \begin{vmatrix} \log\left(r^{p-1-q+1}\right) & (p-q) \\ \log r^{q-1-r+1} & (q-r) \end{vmatrix}$$

$$= \begin{vmatrix} \log r^{p-q} & (p-q) \\ \log r^{q-r} & (q-r) \end{vmatrix}$$

$$= \begin{vmatrix} (p-q)\log r & (p-q) \\ (q-r)\log r & (q-r) \end{vmatrix}$$

= taking $\log r$ common from the C_1.

$$\log r \begin{vmatrix} (p-q) & (p-q) \\ (q-r) & (q-r) \end{vmatrix}$$

$= \log r \cdot 0$

$= 0$ $\quad (\because C_1 = C_2)$

(proved)

#8 $2s = a+b+c$

given show that $\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c)$

Proof:

Let $s-a = u$

$s-b = v$

& $s-c = w$

$\Rightarrow (u+v+w) = 3s-a-b-c$

$\Rightarrow u+v+w = 3s-(a+b+c)$

$\Rightarrow u+v+w = 3s-2s \quad (\because a+b+c = 2s \text{ (given)})$

$\Rightarrow u+v+w = s$

Now $\Rightarrow u+v+w = s$.

$\Rightarrow u = s-v-w \Rightarrow s-a = s-v-w$

$\Rightarrow -a = -v-w$

$\Rightarrow a = v+w$ — (*)

$v = s-u-w$

$\Rightarrow s-b = s-u-w$

$\Rightarrow b = u+w$ — (*)

$w = s-u-v$

$\Rightarrow s-c = s-u-v$

$\Rightarrow c = u+v$ — (*)

P.T.O

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$$\begin{vmatrix} a^2 & (s-a)^2 & (s+a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} (v+w)^2 & u^2 & u^2 \\ v^2 & (w+u)^2 & v^2 \\ w^2 & w^2 & (u+v)^2 \end{vmatrix}$$

$\because$ $C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} (v+w)^2 - u^2 & u^2 - u^2 & u^2 \\ v^2 - (w+u)^2 & (w+u)^2 - v^2 & v^2 \\ w^2 - w^2 & w^2 - (u+v)^2 & (u+v)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (v+w+u)(v+w-u) & 0 & u^2 \\ (v+w+u)(v-w-u) & (w+u+v)(w+u-v) & v^2 \\ 0 & (w+u+v)(w-u-v) & (u+v)^2 \end{vmatrix}$$

$\because$ Taking Common $(u+v+w)$ from C_1 & C_2

$$= (u+v+w)(u+v+w) \begin{vmatrix} v+w-u & 0 & u^2 \\ v-w-u & w+u-v & v^2 \\ 0 & w-u-v & (u+v)^2 \end{vmatrix}$$

~~$R_3 \to R_3 - R_2$~~ ~~$R_2 \to R_2 - R_1$~~ $R_1 \to R_1 - R_2 - R_3$

$$= (u+v+w)^2 \begin{vmatrix} v+w-u-v+w+u & 0-w-u+v-w+u+v & -u^2-v^2-u^2-v^2+2uv \\ v-u-w & w+u-v & v^2 \\ 0 & w-u-v & (u+v)^2 \end{vmatrix}$$

$$= (u+v+w)^2 \begin{vmatrix} 2w & -2w & -2v^2+2uv \\ v-u+w & w+u-v & v^2 \\ 0 & w-u-v & (u+v)^2 \end{vmatrix}$$

$$= (u+v+w)^2 \begin{vmatrix} 2w & -2w & 2(-v^2+uv) \\ v-u-w & w+u-v & v^2 \\ 0 & w-u-v & (u+v)^2 \end{vmatrix}$$

Pattern to proving Q section

$\because$ ~~Taking~~ Common ~~C_1 from R_1~~

$$= 2(u+v+w)^2 \begin{vmatrix} w & -w & uv-v^2 \\ v-u-w & w+u-v & v^2 \\ 0 & w-u-v & (u+v)^2 \end{vmatrix}$$

$\because$

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Q. $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\sin^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$

then find the minimum value of x.

$C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \cos^2 x & 4\sin 2x \\ 2 & 1+\cos^2 x & 4\sin 2x \\ 1 & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$= \begin{vmatrix} 0 & 1 & 0 \\ 2 & 1+\cos^2 x & 4\sin 2x \\ 1 & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

$= -1(2 + 6\sin 2x) - 4\sin 2x)$

$= -1(2 + 2\sin 2x)$

$= -2 - 2\sin 2x$

$= -2(1 + \sin 2x) \quad (\because \sin^2 x \geq 0)$

$= -2(1 + \sin^2 0) \qquad x = 0.$

$= -2(1+0^2)$

$= -2(1)$

$= -2$

The minimum value of x is -2.

Q. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then show that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$.

Proof:

Let $P(n) : A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ be a given statement.

Let $n = 1$

L.H.S of $P(1) = A^1 = A$.

R.H.S of $P(1) = \begin{bmatrix} 1+2\cdot 1 & -4\cdot 1 \\ 1 & 1-2\cdot 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$.

L.H.S of $P(1)$ = R.H.S of $P(1)$.

$\therefore P_1$ is true.

Assume that $P(k)$ is true.

$\Rightarrow$ L.H.S of $P(k)$ = R.H.S of $P(k)$

$A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$

We have to show that $P(k+1)$ is true.

i.e L.H.S of $P(k+1)$ = R.H.S of $P(k+1)$.

$A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix} = \begin{bmatrix} 1+2k+2 & -4k-4 \\ k+1 & 1-2k-2 \end{bmatrix}$

$\Rightarrow \boxed{A^{k+1} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}}$

Now $A^{k+1} = A^k \cdot A^1$

$= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \cdot \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

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$$= \begin{bmatrix} (1+2k)3+(-4k\cdot 1) & (1+2k)(-4)+(-4k)\cdot(-1) \\ k\cdot 3 + (1-2k)\cdot 1 & k\cdot(-4)+(1-2k)\cdot(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k+2k+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$$

∴ L.H.S of P(k+1) = R.H.S of P(k+1).

P(k+1) is true.

Hence P(n) is true.

Ans If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$ where I is a unit matrix of order 2.

Prove $A^2 = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 3\cdot 3 + (-5)\cdot(-4) & -5\cdot 3 + (-5)(2) \\ -4\cdot 3 + 2\cdot(-4) & -4\cdot(-5)+2\cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

Now $A^2 - 5A - 14I$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - 5 \cdot \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20-0 & 24-10-14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

= 0 matrix. (proved)

Ext find x & y if $\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x & 2 \\ 1 & y \end{bmatrix} = \begin{bmatrix} -3 & y \\ -1 & 4 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1\cdot x + -2\cdot 1 & 1\cdot 2 + -2\cdot y \\ 3\cdot x + 2\cdot 1 & 3\cdot 2 + 2\cdot y \end{bmatrix} = \begin{bmatrix} -3 & y \\ -1 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x-2 & 2-2y \\ 3x+2 & 6+2y \end{bmatrix} = \begin{bmatrix} -3 & y \\ -1 & 4 \end{bmatrix}$$

Equating both the matrices.

$\Rightarrow$ $x - 2 = -3$

$\Rightarrow x = -3 + 2$

$\Rightarrow \boxed{x = -1}$

$2 - 2y = y$

$-2y = y - 2$

$\rightarrow -2y = 2$

$\Rightarrow \boxed{y = -1}$ (Ans)

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* Verify that $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation $x^2-(a+d)x+(ad-bc)I=0$, where I is the 2×2 unit matrix.

Sol: We have to show that the matrix A is satisfy the given equation $x^2-(a+d)x+(ad-bc)I=0$

i.e $A^2-(a+d)A+(ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=0$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}-(a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix}+(ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=0$

$\Rightarrow \begin{bmatrix} a\cdot a+b\cdot c & a\cdot b+b\cdot d \\ c\cdot a+d\cdot c & c\cdot b+d\cdot d \end{bmatrix}-\begin{bmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{bmatrix}+\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}-\begin{bmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{bmatrix}+\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

$\Rightarrow a^2+a^2$

$\Rightarrow \begin{bmatrix} a^2+bc-a^2-ad+ad-bc & ab+bd-ab-bd+0 \\ ac+cd-ac-cd+0 & bc+d^2-ad-d^2+ad-bc \end{bmatrix}$

$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$=$ O matrix.

* If $A=\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$ then show that $[AB]^T=B^T\cdot A^T$.

Now $[AB]=\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

$=\begin{bmatrix} 1\cdot 1+2\cdot 2+3(-1) & 1\cdot 2+2\cdot 0+3\cdot 1 \\ 3\cdot 1+(-2)2+1\cdot(-1) & 3\cdot 2+-2\cdot 0+1\cdot 1 \end{bmatrix}$

$=\begin{bmatrix} 1+4-3 & 2+0+3 \\ 3-4-1 & 6-0+1 \end{bmatrix}$

$\Rightarrow [AB]=\begin{bmatrix} 2 & 5 \\ -2 & 7 \end{bmatrix}$

$\Rightarrow [AB]^T=\begin{bmatrix} 2 & -2 \\ 5 & 7 \end{bmatrix}$

Now $B^T=\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

$A^T=\begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \end{bmatrix}$

Now $B^T\cdot A^T=\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \end{bmatrix}$

$=\begin{bmatrix} 1\cdot 1+2\cdot 2+(-1)3 & 1\cdot 3+2\cdot(-2)+(-1)\cdot 1 \\ 2\cdot 1+0(-2)+1\cdot 3 & 2\cdot 3+0(-2)+1\cdot 1 \end{bmatrix}$

$=\begin{bmatrix} 1+4-3 & 3-4-1 \\ 2+0+3 & 6+0+1 \end{bmatrix}$

$\Rightarrow B^T\cdot A^T=\begin{bmatrix} 2 & -2 \\ 5 & 7 \end{bmatrix}$

$\therefore [AB]^T=B^T\cdot A^T$ (proved).

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* Matrices x and y are such that $3x+4y=I$, $x-2y=I$, where I is a 3×3 unit matrix, then determine x & y.

Now $3x+4y=I\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$$x-2y=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Suppose

$$3x+4y=I \quad \text{---(i)}$$
$$x-2y=I \quad \text{---(ii)}$$

eq^n (i) $\times 1 \Rightarrow 3x+4y=I$

eq^n (ii) $\times(2) \Rightarrow 2x-4y=2I$

$$+\quad\overline{}$$

$$5x=3I$$

$$\Rightarrow \boxed{x=\frac{3}{5}I}.$$

Now putting the value of 'x' in eq^n (i)

$$\frac{3}{5}I-2y=I$$
$$\Rightarrow -2y=I-\frac{3}{5}I$$
$$\Rightarrow -2y=\frac{5I-3I}{5}$$
$$\Rightarrow -2y=\frac{2I}{5}$$
$$\Rightarrow y=\frac{2}{5}I\times\frac{1}{-2}$$
$$\Rightarrow \boxed{y=-\frac{1}{5}I}$$

then $x=\frac{3}{5}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \Rightarrow x=\begin{bmatrix}\frac{3}{5}&0&0\\0&\frac{3}{5}&0\\0&0&\frac{3}{5}\end{bmatrix}$

$y=$

$y=-\frac{1}{5}I \Rightarrow y=-\frac{1}{5}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$\Rightarrow y=\begin{bmatrix}-\frac{1}{5}&0&0\\0&-\frac{1}{5}&0\\0&0&-\frac{1}{5}\end{bmatrix}$

Q | If $A=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}$, $B=\begin{bmatrix}3&1&-2\\4&2&5\\2&0&3\end{bmatrix}$

$$C=\begin{bmatrix}4&1&2\\0&3&2\\1&-2&3\end{bmatrix}$$

then show that $A+(B-C)=(A+B)-C$

Now L.H.S

$$\therefore A+(B-C)=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}+\left\{\begin{bmatrix}3&1&-2\\4&2&5\\2&0&3\end{bmatrix}-\begin{bmatrix}4&1&2\\0&3&2\\1&-2&3\end{bmatrix}\right\}$$

$$=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}+\begin{bmatrix}3-4&1-1&-2-2\\4-0&2-3&-2\\2-1&0+2&3-3\end{bmatrix}$$

$$=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}+\begin{bmatrix}-1&0&-4\\4&-1&3\\1&2&0\end{bmatrix}$$

$$=\begin{bmatrix}1-1&2+0&-3-4\\5+4&0-1&2+3\\1+1&-1+2&1+0\end{bmatrix}$$

$$\Rightarrow A+(B-C)=\begin{bmatrix}0&2&-7\\9&-1&5\\2&1&1\end{bmatrix}$$

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Now R.H.S

$(A+B)-C$

$$= \left\{\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}\right\} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 2+1 & -3-2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & -5 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 3-1 & -5-2 \\ 9-0 & 2-3 & 7-2 \\ 3-1 & -1+2 & 4-3 \end{bmatrix}$$

$$\Rightarrow (A+B)-C = \begin{bmatrix} 0 & 2 & -7 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

$\therefore$ L.H.S = R.H.S (proved).

Q | 6 A, B and C are matrices of order 2.

and $2A+B+C = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$, $A+B+C = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$

$A+B-C = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

Given

$2A+B+C = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ — (I)

$A+B+C = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$ — (II)

$A+B-C = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ — (III)

Now add substract eqn (I) − eqn (II).

$2A+B+C = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

$(-)\ (-)\ (-)\quad A+B+C = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$ (−1)

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1-0 & 2-1 \\ 3-2 & 0-1 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$ — (IV)

Now ~~Substract~~ ~~Adding~~ eqn (II) & eqn (III) — eqn (II) + eqn (III)

$A+B+C = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$

$(-)\ (-)\ (+)\quad A+B-C = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

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Now substract eqn (II) − eqn (III).

$$A+B+C=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}$$

$$A+B-C=\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}$$

$$2C=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}-\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}$$

$$\Rightarrow 2C=\begin{bmatrix}0-1 & 1-2\\ 2-1 & 1-0\end{bmatrix}$$

$$\Rightarrow 2C=\begin{bmatrix}-1 & -1\\ 1 & 1\end{bmatrix}$$

$$\Rightarrow C=\frac{\begin{bmatrix}-1 & -1\\ 1 & 1\end{bmatrix}}{2}$$

$$\Rightarrow C=\begin{bmatrix}-1/2 & -1/2\\ 1/2 & 1/2\end{bmatrix} \quad \text{— (IV) (X)}$$

Now putting the value of A & C in eqn (I)

$$\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}+B+\begin{bmatrix}-1/2 & -1/2\\ 1/2 & 1/2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1-1/2 & 1-1/2\\ 1+1/2 & -1+1/2\end{bmatrix}+B=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1/2 & 1/2\\ 3/2 & -1/2\end{bmatrix}+B=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}$$

$$\Rightarrow B=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}-\begin{bmatrix}1/2 & 1/2\\ 3/2 & -1/2\end{bmatrix}$$

$$\Rightarrow B=\begin{bmatrix}0-1/2 & 1-1/2\\ 2-3/2 & 1+1/2\end{bmatrix}$$

$$\Rightarrow B=\begin{bmatrix}-1/2 & 1/2\\ 1/2 & 3/2\end{bmatrix} \quad \text{— (I)}$$

Theorem :- 1 :-

For any Square matrix A.

(i) $A+A^T$ is a Symmetric matrix.

(ii) $A-A^T$ is a Skew Symmetric matrix.

$$(A+B)^T=A^T+B^T$$

$$\Rightarrow A=\begin{bmatrix}a_1 & b_1\\ a_2 & b_2\end{bmatrix} \quad B=\begin{bmatrix}\alpha_1 & \beta_1\\ \alpha_2 & \beta_2\end{bmatrix}$$

$$\Rightarrow (A+B)=\begin{bmatrix}a_1+\alpha_1 & b_1+\beta_1\\ a_2+\alpha_2 & b_2+\beta_2\end{bmatrix}$$

$$\Rightarrow \underline{(A+B)^T}=\begin{bmatrix}a_1+\alpha_1 & a_2+\alpha_2\\ b_1+\beta_1 & b_2+\beta_2\end{bmatrix} \quad \text{— (X)}$$

$$A^T=\begin{bmatrix}a_1 & a_2\\ b_1 & b_2\end{bmatrix} \quad B^T=\begin{bmatrix}\alpha_1 & \alpha_2\\ \beta_1 & \beta_2\end{bmatrix}$$

$$\underline{A^T+B^T}=\begin{bmatrix}a_1+\alpha_1 & a_2+\alpha_2\\ b_1+\beta_1 & b_2+\beta_2\end{bmatrix} \quad \text{— (+)}$$

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* Symmetric Matrix :-

→ A Square Matrix "A" is said to be symmetric matrix if $A = A^T$

* Skew-Symmetric Matrix :-

→ A Square Matrix "A" is said to be skew matrix if $A = -A^T$

Theorem - 1 :-

Prove (i) $A + A^T$ is a Symmetric matrix.

$A + A^T$ is Symmetric

i.e we have to show that $(A + A^T) = (A + A^T)^T$

Now $(A + A^T)^T = A^T + (A^T)^T$ $\quad (\because (A+B)^T = A^T + B^T)$

$= A^T + A$ $\quad (\because (A^T)^T = A)$

$= A + A^T$

$(A + A^T) = (A + A^T)^T$

$\therefore (A + A^T)$ is a Symmetric. (proved)

Prove (ii) $A - A^T$ is a skew symmetric matrix.

$A - A^T$ is a skew symmetric matrix.

i.e we have to show that $(A - A^T) = -(A - A^T)^T$

=

~~Now $-(A - A^T)^T$~~

~~$= (A^T - A)^T$~~

~~$= (A^T)^T + A^T$~~

~~$= -(A + A^T)$~~

~~$= -(A^T + A)$~~

Now $-(A - A^T)^T$ $\quad (\because (A+B)^T = A^T + B^T)$

$= -A^T + (A^T)^T$

$= -A^T + A$ $\quad (\because (A^T)^T = A)$

$= A - A^T$

$\therefore (A - A^T) = -(A - A^T)^T$

$\therefore (A - A^T)$ is a skew symmetric matrix.

* let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

→ Minor of $a_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

→ Minor of $a_{12} = M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

→ Minor of $a_{13} = M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

→ Minor of $a_{21} = M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$

→ Minor of $a_{22} = M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$

→ Minor of $a_{23} = M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$

→ Minor of $a_{31} = M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$

Skew $\begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & 3 \\ -4 & -3 & 0 \end{bmatrix}$

$\begin{pmatrix} 0 & 2 & 4 \\ 2 & 0 & 3 \\ 4 & 3 & 0 \end{pmatrix}$ Sym

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→ Minor of a_{32} (M_{32}) $= \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$

→ Minor of a_{33} (M_{33}) $= \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

Let A be a Square matrix

i.e $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

then determinant of $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

the minor of $a_{ij} = M_{ij}$ is the determinant obtained by eliminating the i^{th} row and j^{th} column from the determinant A.

☆ Co-Factor :-

→ The Co-factor of a_{ij} is denoted by C_{ij} and

$$C_{ij} = (-1)^{i+j} M_{ij}$$

→ Minor of $a_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{33} \cdot a_{32}$

Co-factor of $a_{11} = (-1)^{1+1} \cdot M_{11} = (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

$\Rightarrow C_{11} = 1(a_{22} \cdot a_{33} - a_{23} \cdot a_{32})$

→ Minor of $a_{12} = M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = a_{21} \cdot a_{33} - a_{23} \cdot a_{31}$

Co factor of $a_{12} = (-1)^{1+2} \cdot M_{12} = (-1)^3 \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

$\Rightarrow C_{12} = -1(a_{21} \cdot a_{33} - a_{23} \cdot a_{31})$

$\Rightarrow C_{12} = (a_{23} \cdot a_{31} - a_{21} \cdot a_{33})$

→ Minor of $a_{13} = M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = a_{21} \cdot a_{32} - a_{22} \cdot a_{31}$

Cofactor of $a_{13} = C_{13} = (-1)^{1+3} \cdot M_{13} = (-1)^4 \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$= 1 \cdot (a_{21} \cdot a_{32} - a_{22} \cdot a_{31})$

→ Minor factor of $a_{21} = M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = a_{12} \cdot a_{33} - a_{32} \cdot a_{13}$

Co-factor of $a_{21} = C_{21} = (-1)^{2+1} \cdot M_{21} = (-1)^3 \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$

$= (-1)(a_{12} \cdot a_{33} - a_{13} \cdot a_{32})$

$= (a_{13} \cdot a_{32} - a_{12} \cdot a_{33})$

→ Minor factor of $a_{22} = M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} = a_{11} \cdot a_{33} - a_{13} \cdot a_{31}$

Co-factor of $a_{22} = C_{22} = (-1)^{2+2} \cdot M_{22} = (-1)^4 (a_{11} \cdot a_{33} - a_{13} \cdot a_{31})$

$= 1 \cdot (a_{11} \cdot a_{33} - a_{13} \cdot a_{31})$

→ Minor factor of $a_{23} = M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} = a_{11} \cdot a_{32} - a_{12} \cdot a_{31}$

Co-factor of $a_{23} = C_{23} = (-1)^{2+3} \cdot M_{23} = (-1)^{2+3} (a_{11} \cdot a_{32} - a_{12} \cdot a_{31})$

$= -1(a_{11} \cdot a_{32} - a_{12} \cdot a_{31})$

$= -(a_{12} \cdot a_{31} - a_{11} \cdot a_{32})$

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→ Minor factor of $a_{31} = M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} = a_{12}\cdot a_{23} - a_{22}\cdot a_{13}$.

→ Co-factor of $a_{31} = C_{31} = (-1)^{3+1}\cdot M_{31} = (-1)^4 (a_{12}\cdot a_{23} - a_{22}\cdot a_{13})$
$= 1\,(a_{12}\cdot a_{23} - a_{22}\cdot a_{13})$

→ Minor factor of $a_{32} = M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = (a_{11}\cdot a_{23} - a_{13}\cdot a_{21})$

Cofactor of $a_{32} = C_{32} = (-1)^{3+2}\cdot M_{32} = (-1)^5 (a_{11}\cdot a_{23} - a_{13}\cdot a_{21})$

→ Minor factor of $a_{33} = M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (a_{13}\cdot a_{21} - a_{11}\cdot a_{23})$
$= (a_{11}\cdot a_{22} - a_{21}\cdot a_{12})$

Co-factor of $a_{33} = C_{33} = (-1)^{3+3}\cdot M_{33} = 1\cdot(a_{11}\cdot a_{22} - a_{21}\cdot a_{12})$

★ Adjoint of a matrix:-

→ Let A be a square matrix of order n and C_{ij} ; the co-factors of the elements a_{ij} then the transpose of the Co-factor matrix of "A" is called the adjoint of A.

eg Let $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 4 \\ 5 & 6 \end{vmatrix}$.

Minor of $a_{11}(2) = M_{11} = 6$, Co-factor of $a_{11}(2) = (-1)^{1+1}\cdot M_{11} = (-1)^2\cdot 6 = 6$

Minor of $a_{12}(4) = M_{12} = 5$, Co-factor of $a_{12}(4) = (-1)^{1+2}\cdot M_{12} = (-1)^3\cdot 5 = -5$

Minor of $a_{21}(5) = M_{21} = 4$, Co-factor of $a_{21}(5) = (-1)^{2+1}\cdot M_{21} = (-1)^3\cdot 4 = -4$.

Minor of $a_{22}(6) = M_{22} = 2$, Co-factor of $a_{22}(6) = (-1)^{2+2}\cdot 2 = 2(-1)^4\cdot 2 = 2$

i.e $C_{11} = 6 \quad C_{12} = -5$
$C_{21} = -4 \quad C_{22} = 2$

Co-factor matrix of $[A] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ -4 & 2 \end{bmatrix}$

adjoint of $A = \begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix} = \begin{bmatrix} 6 & -4 \\ -5 & 2 \end{bmatrix}$

Note:-
→ If A is square matrix of order 'n'
then adjoint of A is also a square matrix of order 'n'.

★ Inverse of a Matrix:-

→ If A is a square matrix, there exist a matrix B such that $AB = BA = I$.

→ Then B is called inverse matrix of A.
★ Inverse of the matrix A is denoted by A^{-1}.
Hence $AA^{-1} = A^{-1}A = I$.

★ Note:-
→ The order of identy matrix I is same as order of matrix 'A'.
→ Order of 'A^{-1}' is same as order of matrix 'A'.

★ How to find inverse of a Square matrix:-

If A is a non-singular square matrix (i.e $|A| \neq 0$)
then $A^{-1} = \frac{\text{adj}\,A}{|A|}$

eg If $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 3 \\ 4 & 5 \end{vmatrix} = 5 - 12 = -7 \neq 0$

So A is non-singular matrix.

→ Minor factor of $1 = 5$
Co-factor of $1\,(a_{11}) = (-1)^{1+1}\cdot M_{11} = +a(-1)^2\cdot 5 = 5 = C_{11}$

→ Misc

Inverse of a symmetric matrix → Symmetric.

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→ Minor of 3 (M_{12}) = 4, Co factor of $3 = (-1)^{1+2} \cdot M_{12} = -1 \cdot 4 = -4 = C_{12}$

→ Minor factor of 4 (M_{21}) = 3, co-factor of $4 = (-1)^{2+1} \cdot M_{21} = -1 \cdot 3 = -3 = C_{21}$

→ Minor factor of 5 (M_{22}) = 1, co-factor of $5 = (-1)^{2+2} \cdot M_{22} = +1 \cdot 1 = 1 = C_{22}$

then the co-factor Matrix of $A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

$$= \begin{bmatrix} 5 & -4 \\ -3 & 1 \end{bmatrix}$$

adjoint of A = (cofactor of matrix)^T

$$= \begin{bmatrix} 5 & -3 \\ -4 & 1 \end{bmatrix}$$

Now $A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 5 & -3 \\ -4 & 1 \end{bmatrix}}{-7} = \begin{bmatrix} -5/7 & 3/7 \\ 4/7 & -1/7 \end{bmatrix}$

Now $A \cdot A^{-1} = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} -5/7 & 3/7 \\ 4/7 & -1/7 \end{bmatrix}$

$$= \begin{bmatrix} -5/7 + 12/7 & 3/7 - 3/7 \\ -20/7 + 20/7 & 12/7 - 5/7 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{12-5}{7} & \frac{0}{7} \\ \frac{-20+20}{7} & \frac{12-5}{7} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{7} & 0 \\ 0 & \frac{7}{7} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \text{unit matrix of order 2. (proved)}$$

$A \cdot A^{-1} = $ Unit matrix

★ Solution of System of linear eqn by Matrix inverse method:-

→ ~~Let us~~

→ Let us consider a system of linear equation of 3 variable.

Suppose:

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right\} \quad \text{(1)}$$

This equation can be written as

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}_{3\times3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}_{3\times1}$$

Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

⇒ ~~AOX~~

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$AX = B$

$\Rightarrow X = A^{-1}B$

$\because A^{-1}(AX) = A^{-1}B$

$(A^{-1}A)X = A^{-1}B$

$\Rightarrow IX = A^{-1}B.$

$\Rightarrow X = A^{-1}B$ (Ans)

$A(BC) = (AB)C.$

eg:- Solve the equation by Matrix Method =

→ $x + 2y = 1$

$3x + 4y = 4$

Sol^n

We have $x + 2y = 1$

$3x + 4y = 4$

The above equation can be written as

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$\Rightarrow AX = B$, where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$\Rightarrow \boxed{X = A^{-1}B}$

Now $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2 \neq 0$

so A^{-1} exist. & A is a non-singular matrix.

Minor of $1\,(M_{11}) = 4$, Cofactor of $1\,(C_{11}) = (-1)^{1+1} M_{11} = 1 \cdot 4 = 4$.

Minor of $2\,(M_{12}) = 3$, Cofactor of $2\,(C_{12}) = (-1)^{1+2} M_{12} = -1 \cdot 3 = -3$.

Minor of $3\,(M_{21}) = 2$, Cofactor of $3\,(C_{21}) = (-1)^{2+1} \cdot M_{21} = -1 \cdot 2 = -2$.

Minor of $4\,(M_{22}) = 1$, Cofactor of $4\,(C_{22}) = (-1)^{2+2} M_{22} = 1 \cdot 1 = 1$.

Cofactor matrix of $A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

$= \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$

Now adj of A = transpose (Cofactor matrix)^T

$= \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$.

Now $A^{-1} = \dfrac{\text{Adj}(A)}{|A|}$

$= \dfrac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{-2} = \begin{bmatrix} -4/2 & 2/2 \\ 3/2 & -1/2 \end{bmatrix}$

We have

$X = A^{-1}B.$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4/2 & 2/2 \\ 3/2 & -1/2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4/2 \cdot 1 + 2/2 \cdot 4 \\ 3/2 \cdot 1 + -1/2 \cdot 4 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-4+4}{2} \\ \frac{3-4}{2} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -1/2 \end{bmatrix}$ $\Rightarrow$ Comparing both the equation

$\boxed{x = 0 \text{ & } y = -1/2}$ Ans

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Solve the following eqn by matrix method:

$$\left.\begin{aligned} 2x - y + z &= 4 \\ x + 3y + 2z &= 12 \\ 3x + 2y + 3z &= 16 \end{aligned}\right\} \longrightarrow (*)$$

Soln

We have $\left.\begin{aligned} 2x - y + z &= 4 \\ x + 3y + 2z &= 12 \\ 3x + 2y + 3z &= 16 \end{aligned}\right\} \longrightarrow (*)$

The above eqn can be written as

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & 2 \\ 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 16 \end{bmatrix}$$

$AX = B$, where $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & 2 \\ 3 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 4 \\ 12 \\ 16 \end{bmatrix}$

$X = A^{-1}B$.

Now $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & 2 \\ 3 & 2 & 3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix}$$

$= 2(9-4) + 1(3-6) + 1(2-9)$

$= 10 - 3 - 7$

$= 10 - 10$

$= 0 \Rightarrow |A| = 0$ (∵ does not exist)

A

(11) by Matrix method:-

$$\left.\begin{aligned} 2x + 3y + 3z &= 5 \\ x - 2y + z &= -4 \\ 3x - y - 2z &= 3 \end{aligned}\right\}$$

Soln We have

$$\left.\begin{aligned} 2x + 3y + 3z &= 5 \\ x - 2y + z &= -4 \\ 3x - y - 2z &= 3 \end{aligned}\right\} \longrightarrow (*)$$

The above eqn can be written as

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$AX = B$, where $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$X = A^{-1}B$.

Now $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 2(4+1) - 3(-2-3) + 3(-1+6)$$

$= 2(-6-2) - 3(-3-3) + 3(-1+6)$

$= -16 + 24$

$= 8$

$= 20 + 15 + 15$

$\Rightarrow |A| = 40 \neq 0$

So A^{-1} exist.

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Minor of 2 (M_{11}) = 5, Cofactor of 2 (C_{11}) = $(-1)^{1+1} \cdot M_{11} = 1 \cdot 5 = 5$.

Minor of 3 (M_{12}) = −5, Cofactor of 3 (C_{12}) = $(-1)^{1+2} \cdot M_{12} = -1 \cdot -5 = 5$

Minor of 3 (M_{13}) = 5, Cofactor of 3 (C_{13}) = $(-1)^{1+3} \cdot M_{13} = 1 \cdot 5 = 5$

Minor of 1 (M_{21}) = −3, Cofactor of 1 (C_{21}) = $(-1)^{2+1} \cdot M_{21} = -1 \cdot -3 = 3$

Minor of −2 (M_{22}) = −13, Cofactor of −2 (C_{22}) = $(-1)^{2+2} \cdot M_{22} = -1 \cdot -13 = -13$

Minor of 1 (M_{23}) = −11, Co-factor of 1 (C_{23}) = $(-1)^{2+3} \cdot M_{23} = -1 \cdot -11 = 11$

Minor of 3 (M_{31}) = 9, Cofactor of 3 (C_{31}) = $(-1)^{3+1} \cdot M_{31} = 1 \cdot 9 = 9$

Minor of −1 (M_{32}) = −1, Co-factor of −1 (C_{32}) = $(-1)^{3+2} \cdot M_{32} = -1 \cdot -1 = 1$.

Minor of −2 (M_{33}) = −7, Co-factor of −7 (C_{33}) = $(-1)^{3+3} \cdot M_{33} = 1 \cdot -7 = -7$

Co-factor matrix of $A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

$= \begin{bmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{bmatrix}$

adj. of A = (Co-factor matrix)^T

$= \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$

Now $A^{-1} = \frac{\text{adj } A}{|A|}$

$= \frac{\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}}{40}$

$A^{-1} = \begin{bmatrix} 1/8 & 3/40 & 9/40 \\ 1/8 & -13/40 & 1/40 \\ 1/8 & 11/40 & -7/40 \end{bmatrix}$

$\Rightarrow$ We have $X = A^{-1}B$.

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/8 & 3/40 & 9/40 \\ 1/8 & -13/40 & 1/40 \\ 1/8 & 11/40 & -7/40 \end{bmatrix}_{3\times3} \cdot \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}_{3\times1}$

$= \frac{1}{8} \cdot 5$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5/8 - \frac{12}{40} + \frac{27}{40} \\ 5/8 + \frac{52}{40} + \frac{3}{40} \\ 5/8 - \frac{44}{40} - \frac{21}{40} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{25-12+27}{40} \\ \frac{25+52+3}{40} \\ \frac{25-44-21}{40} \end{bmatrix} = \begin{bmatrix} \frac{40}{40} \\ \frac{80}{40} \\ \frac{-40}{40} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

Comparing both the eqn.

$x = 1$, $y = 2$ & $z = -1$.

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Solve the equation by matrix method. (1,21)

$2x + y - 4z = 1$

$3x + 6y + 9z = 4$

$x + 3y + 2z = 6$

Sol: We have

$$\left.\begin{aligned} 2x+y-4z&=1\\ 3x+6y+9z&=4\\ x+3y+2z&=6 \end{aligned}\right\} \text{---} (*)$$

The above equation can be written as

$$\begin{bmatrix} 2 & 1 & -4 \\ 3 & 6 & 9 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$$

$AX = B$, where $A = \begin{bmatrix} 2 & 1 & -4 \\ 3 & 6 & 9 \\ 1 & 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$

$X = A^{-1}B$

Now $A = \begin{bmatrix} 2 & 1 & -4 \\ 3 & 6 & 9 \\ 1 & 3 & 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 1 & -4 \\ 3 & 6 & 9 \\ 1 & 3 & 2 \end{vmatrix}$$

$= 2(12-27) - 1(6-9) - 4(9-6)$

$= -30 + 3 - 12$

$= -39 \neq 0$

So, A^{-1} is exist.

Minor of $2(M_{11}) = -15$, Cofactor of $2 = (-1)^{1+1} \cdot M_{11} = 1 \cdot -15 = -15$

Minor of $1(M_{12}) = -3$, Cofactor of $1 = (-1)^{1+2} \cdot M_{12} = -1 \cdot -3 = 3$.

Minor of $-4(M_{13}) = 3$, Cofactor of $-4 = (-1)^{1+3} \cdot M_{13} = 1 \cdot 3 = 3$.

Minor of $3(M_{21}) = 14$ Cofactor of $3 = (-1)^{2+1} \cdot M_{21} = -1 \cdot 14 = -14$

Minor of $6(M_{22}) = 8$, Cofactor of $6 = (-1)^{2+2} \cdot M_{22} = 1 \cdot 8 = 8$

Minor of $9(M_{23}) = 5$, Cofactor of $9 = (-1)^{2+3} \cdot M_{23} = -1 \cdot 5 = -5$

Minor of $1(M_{31}) = 33$, Cofactor of $1 = (-1)^{3+1} \cdot M_{31} = 1 \cdot 33 = 33$.

Minor factor of $3(M_{32}) = 30$, Cofactor of $3 = (-1)^{3+2} \cdot M_{32} = -1 \cdot 20 = -20$.

Minor factor of $2(M_{33}) = 9$, Cofactor of $2 = (-1)^{3+3} \cdot M_{33} = 1 \cdot 9 = 9$.

Cofactor matrix of $A = \begin{bmatrix} -15 & 3 & 3 \\ -14 & 8 & -5 \\ 33 & -20 & 9 \end{bmatrix}$

$$\text{Adj}(A) = \begin{bmatrix} -15 & -14 & 33 \\ 3 & 8 & -20 \\ 3 & -5 & 9 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} -15 & -14 & 33 \\ 3 & 8 & -20 \\ 3 & -5 & 9 \end{bmatrix}}{-39}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 15/39 & 14/39 & -33/39 \\ -1/13 & -8/39 & +20/39 \\ -1/13 & +5/39 & -9/39 \end{bmatrix}$$

$\Rightarrow A^{-1} = \begin{bmatrix}$

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$$A^{-1} = \begin{bmatrix} 15/39 & 14/39 & -33/39 \\ -1/13 & -8/39 & 20/39 \\ -1/13 & 5/39 & -9/39 \end{bmatrix}$$

we have

$X = A^{-1} \cdot B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15/39 & 14/39 & -33/39 \\ -1/13 & -8/39 & 20/39 \\ -1/13 & 5/39 & -9/39 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15/39 + 56/39 + (-198/39) \\ -1/13 + -32/39 + 180/39 \\ -1/13 + 20/39 + -54/39 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{15+56-198}{39} \\ \frac{-3-32+180}{39} \\ \frac{-3+20-54}{39} \end{bmatrix} = \begin{bmatrix} -87/39 \\ 145/39 \\ -37/39 \end{bmatrix}$$

$\Rightarrow x = \frac{-87}{39}$, $y = \frac{145}{39}$, $z = \frac{-37}{39}$.

Solve the following Equations by Inverse Matrix Method

(i) $x + y + z = 6$
$x - y + 2z = 5$
$3x + y + z = 8$

Sol^n

we have.

$$\left.\begin{aligned} x + y + z &= 6 \\ x - y + 2z &= 5 \\ 3x + y + z &= 8 \end{aligned}\right\} \quad \text{---} (*)$$

The above equation can be written as.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 8 \end{bmatrix}$$

Here, $AX = B$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 6 \\ 5 \\ 8 \end{bmatrix}$

$X = A^{-1}B$.

Now $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = 1(-1-2) - 1(1-6) + 1(4)$

$= -3 + 5 + 4 = 6$

$\Rightarrow \boxed{|A| = 6}$.

$M_{11} = 1$

Minor of 1 $(M_{11}) = -3$, Cofactor of 1 $(C_{11}) = (-1)^{1+1} \cdot M_{11} = 1 \cdot -3 = -3$

Minor of 1 $(M_{12}) = -5$, Cofactor of 1 $(C_{12}) = (-1)^{1+2} \cdot M_{12} = -1 \cdot -5 = 5$

Minor of 1 $(M_{13}) = 4$, Cofactor of 1 $(C_{13}) = (-1)^{1+3} \cdot M_{13} = 1 \cdot 4 = 4$.

Minor of 1 $(M_{21}) = 0$, Cofactor of 1 $(B_2 C_{21}) = (-1)^{2+1} \cdot M_{21} = 0 \cdot -1 \cdot 0 = 0$.

Minor of -1 $(M_{22}) = -2$, Cofactor of -1, $(C_{22}) = (-1)^{2+2} \cdot M_{22} = 1 \cdot -2 = -2$

Minor of 2 $(M_{23}) = -2$, Cofactor of -2 $(C_{23}) = (-1)^{2+3} \cdot M_{23} = -1 \cdot -2 = 2$

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→ Minor of 3 (M_{31}) = 3, Cofactor of 3 (C_{31}) = $(-1)^{3+1} \cdot M_{31} = 1 \cdot 3 = 3$

→ Minor of 1 (M_{32}) = 1, Cofactor of 1 (C_{32}) = $(-1)^{3+2} \cdot M_{32} = -1 \cdot 1 = -1$

→ Minor of -1 (M_{33}) = -2, Cofactor of -1 (C_{33}) = $(-1)^{3+3} \cdot M_{33} = 1 \cdot -2 = -2$

Now Cofactor matrix of $A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

$= \begin{bmatrix} -3 & 5 & 4 \\ 0 & -2 & 2 \\ 3 & -1 & -2 \end{bmatrix}$

Now Adjoint of A = [Co-factor matrix]^T

$= \begin{bmatrix} -3 & 0 & 3 \\ 5 & -2 & -1 \\ 4 & 2 & -2 \end{bmatrix}$

Now $A^{-1} = \frac{Adj\,A}{|A|} = \frac{\begin{bmatrix} -3 & 0 & 3 \\ 5 & -2 & -1 \\ 4 & 2 & -2 \end{bmatrix}}{6}$

$\Rightarrow A^{-1} = \begin{bmatrix} -3/6 & 0 & 3/6 \\ 5/6 & -2/6 & -1/6 \\ 4/6 & 2/6 & -2/6 \end{bmatrix}$

$\Rightarrow$ Now $X = A^{-1}B$.

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3/6 & 0 & 3/6 \\ 5/6 & -2/6 & -1/6 \\ 4/6 & 2/6 & -2/6 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 5 \\ 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -16/6 + 0 + 24/6 \\ 30/6 - 10/6 - 8/6 \\ 24/6 + 10/6 - 16/6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 + 4 \\ \frac{30 - 10 - 8}{6} \\ \frac{24 + 10 - 16}{6} \end{bmatrix} = \begin{bmatrix} 1 \\ 2\frac{12}{6} \\ 3\frac{18}{6} \end{bmatrix}$

→ Now comparing both the matrix:

$x = 1$, $y = 2$ & $z = 3$. (Ans).

(iv) $x + 2y + 3z = 4$

$2x + 4y + 5z = 1$

$3x + 5y + 6 = 2$.

The above equation can be written as.

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

Here $AX = B$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ & $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix} = 1(24 - 25) - 2(12 - 15) + 3(10 - 12)$

$= -1 + 6 - 6$

$= -1$

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Minor of $1\,(M_{11}) = -1$, cofactor of $1 = (-1)^{1+1} \cdot M_{11} = 1 \cdot -1 = -1$

Minor of $2\,(M_{12}) = -3$, cofactor of $2 = (-1)^{1+2} \cdot M_{12} = -1 \cdot -3 = 3$

Minor of $3\,(M_{13}) = -2$, cofactor of $3 = (-1)^{1+3} \cdot M_{13} = 1 \cdot -2 = -2$.

Minor of $2\,(M_{21}) = -3$, cofactor of $2 = (-1)^{2+1} \cdot M_{21} = -1 \cdot -3 = 3$

Minor of $4\,(M_{22}) = -3$, cofactor of $4 = (-1)^{2+2} \cdot M_{22} = 1 \cdot -3 = -3$

Minor of $5\,(M_{23}) = -1$, cofactor of $5\,(C_{23}) = (-1)^{2+3} \cdot M_{23} = -1 \cdot -1 = 1$

Minor of $3\,(M_{31}) = -2$, cofactor of $3\,(C_{31}) = (-1)^{3+1} \cdot M_{31} = 1 \cdot -2 = -2$

Minor of $5\,(M_{32}) = -1$, cofactor of $5\,(C_{32}) = (-1)^{3+2} \cdot M_{32} = -1 \cdot -1 = 1$

Minor of $6\,(M_{33}) = 0$, cofactor of $6\,(C_{33}) = (-1)^{3+3} \cdot 6 = 1 \cdot 0 = 0$

Cofactor matrix of $A = \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$

$$\text{Adj. } A = \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}.$$

Now $A^{-1} = \dfrac{\text{Adj } A}{|A|} = \dfrac{\begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}}{-1}$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

Now $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 - 3 + 4 \\ -12 + 3 - 2 \\ 8 - 1 + 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -11 \\ 7 \end{bmatrix}.$$

Comparing both the matrix

$x = 5$, $y = -11$ & $z = 7$ (Ans)

(v) $2x + y + 4z = 2$

$3x + 6y + 9z = 4$

$x + 3y + 2z = 6$

Sol^n We have

$$\left.\begin{aligned} 2x + y + 4z &= 2 \\ 3x + 6y + 9z &= 4 \\ x + 3y + 2z &= 6. \end{aligned}\right\} \text{(A)}$$

The above eq^n can be written as.

$$\begin{bmatrix} 2 & 1 & 4 \\ 3 & 6 & 9 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 & 4 \\ 3 & 6 & 9 \\ 1 & 3 & 2 \end{vmatrix} = 2(12 - 27) - 1(6 - 9) + 4(9 - 6)$$

$$= 2 \cdot (-15) - 1(-3) + 4 \cdot (3).$$

$$= -30 + 3 + 12$$

$$\Rightarrow |A| = -15$$

Minor of $2\,(M_{11}) = -15$, cofactor of $2\,(C_{11}) = (-1)^{1+1} \cdot M_{11} = 1 \cdot -15 = -15$

Minor of $1\,(M_{12}) = -3$, cofactor of $1\,(C_{12}) = (-1)^{1+2} \cdot M_{12} = -1 \cdot -3 = 3$

Minor of $4\,(M_{13}) = 3$, Co-factor of $4\,(C_{13}) = (-1)^{1+3} M_{13} = 1 \cdot 3 = 3$

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Minor of 3 $(M_{21}) = -10$, Cofactor of 3 $(C_{21}) = (-1)^{2+1} \cdot M_{21} = -1 \cdot (-10) = 10$

Minor of 6 $(M_{22}) = 0$, Cofactor of 6 $(C_{22}) = (-1)^{2+2} \cdot M_{22} = 1 \cdot 0 = 0$

Minor of 9 $(M_{23}) = 5$, Co-factor of 9 $(C_{23}) = (-1)^{2+3} \cdot M_{23} = -1 \cdot 5 = -5$

Minor of 1 $(M_{31}) = -15$, Cofactor of 1 $(C_{31}) = (-1)^{3+1} M_{31} = 1 \cdot -15 = -15$

Minor of 3 $(M_{32}) = 6$, Co-factor of 3 $(C_{32}) = (-1)^{3+2} \cdot M_{32} = -1 \cdot 6 = -6$

Minor of 9, $(M_{33}) = 9$, Co-factor of 2 $(C_{33}) = (-1)^{3+3} \cdot M_{33} = 1 \cdot 9 = 9$.

Now Cofactor matrix of $A = \begin{bmatrix} -15 & 3 & 3 \\ 10 & 0 & -5 \\ -15 & -6 & 9 \end{bmatrix}$

Adj. A of $A = \begin{bmatrix} -15 & 10 & -15 \\ 3 & 0 & -6 \\ 3 & -5 & 9 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj.}A}{|A|} = \frac{\begin{bmatrix} -15 & 10 & -15 \\ 3 & 0 & -6 \\ 3 & -5 & 9 \end{bmatrix}}{-15}$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -10/15 & 1 \\ 3/-15 & 0 & -6/15 \\ -3/15 & 5/15 & -9/15 \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -2/3 & 1 \\ -1/5 & 0 & 2/5 \\ -1/5 & 1/3 & -3/5 \end{bmatrix}$

Now $AX = A^{-1}B$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -2/3 & 1 \\ -1/5 & 0 & 2/5 \\ -1/5 & 1/3 & -3/5 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 - 8/3 + 6 \\ -2/5 + 0 + 12/5 \\ -2/5 + 4/3 - 18/5 \end{bmatrix} = \begin{bmatrix} \frac{6 - 8 + 18}{3} \\ \frac{-2 + 12}{5} \\ \frac{-6 + 20 - 54}{15} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16/3 \\ 10/5 \\ -8/3 \end{bmatrix} = \begin{bmatrix} 16/3 \\ 2 \\ -8/3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16/3 \\ 2 \\ -8/3 \end{bmatrix}$

On Comparing both the matrix

$x = 16/3$, $y = 2$, $z = -8/3$. (Ans)

vi) $x - y + z = y$

$2x + y - 3z = 0$

$x + y + z = 2$.

Sol^n We have

$x - y + z = y$

$2x + y - 3z = 0$ } ——— (1)

$x + y + z = 2$.

The above eqn can be written as

$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ 0 \\ 2 \end{bmatrix}$.

where $AX = B$, where $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} y \\ 0 \\ 2 \end{bmatrix}$ & $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$|A| = 1(1+3) + 1(2+3) + 1(2-1)$

$= 4 + 5 + 1$

$\Rightarrow |A| = 10$

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Minor of $1(M_{11}) = 4$, Cofactor of $1(C_{11}) = (-1)^{1+1} \cdot M_{11} = 1 \cdot 4 = 4$

Minor of $-1(M_{12}) = 5$, Cofactor of $-1(C_{12}) = (-1)^{1+2} \cdot M_{12} = -1 \cdot 5 = -5$

Minor of $1(M_{13}) = 1$, Cofactor of $1(C_{13}) = (-1)^{1+3} \cdot M_{13} = 1 \cdot 1 = 1$

Minor of $2(M_{21}) = -2$, Cofactor of $2(C_{21}) = (-1)^{2+1} \cdot M_{21} = -1 \cdot -2 = 2$.

Minor of $1(M_{22}) = 0$, Cofactor of $1(C_{22}) = (-1)^{2+2} \cdot M_{22} = 1 \cdot 0 = 0$

Minor of $-3(M_{23}) = 2$, Cofactor of $-3(C_{23}) = (-1)^{2+3} \cdot M_{23} = -1 \cdot 2 = -2$

Minor of $1(M_{31}) = 2$, Cofactor of $1(C_{31}) = (-1)^{3+1} \cdot M_{31} = 1 \cdot 2 = 2$

Minor of $1(M_{32}) = -5$, Cofactor of $1(C_{32}) = (-1)^{3+2} \cdot M_{32} = -1 \cdot -5 = 5$

Minor of $1(M_{33}) = 3$, Cofactor of $1(C_{33}) = (-1)^{3+3} \cdot M_{33} = 1 \cdot 3 = 3$

Co-factor matrix of $A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$

$$Adj(A) = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{Adj(A)}{|A|} = \frac{\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}}{10}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 4/10 & 2/10 & 2/10 \\ -5/10 & 0 & 5/10 \\ 1/10 & -2/10 & 3/10 \end{bmatrix} = \begin{bmatrix} 2/5 & 1/5 & 1/5 \\ -1/2 & 0 & 1/2 \\ 1/10 & -1/5 & 3/10 \end{bmatrix}$$

We know $X = A^{-1}B$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2/5 & 1/5 & 1/5 \\ -1/2 & 0 & 1/2 \\ 1/10 & -1/5 & 3/10 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8/5 + 0 + 2/5 \\ -4/2 + 0 + 2/2 \\ -4/10 + 0 + 6/10 \end{bmatrix} = \begin{bmatrix} \frac{8+2}{5} \\ -2+1 \\ \frac{-4+6}{10} \end{bmatrix} = \begin{bmatrix} \frac{10}{5} \\ -1 \\ \frac{10}{10} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Comparing with the matrix

$x = 2$, $y = -1$ & $z = 1$. Ans.

(vii) $x + y + z = 2$

$2x + y + z = 4$

$x + y - z = 1$

First we have

$$\left.\begin{aligned} x + y + z &= 2 \\ 2x + y + z &= 4 \\ x + y - z &= 1 \end{aligned}\right\} \text{---- (A)}$$

The above eqn can be written as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

~~AX = B~~. Here $AX = B$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 1(-1-1) - 1(-2-1) + 1(2-1)$$

$$= 1(-2) - 1(-3) + 1(1)$$

$$= -2 + 3 + 1$$

$$\Rightarrow |A| = 2$$

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Minor of $1(M_{11}) = -2$, Co-factor of $1(C_{11}) = (-1)^{1+1} \cdot M_{11} = 1 \cdot -2 = -2$

Minor of $1(M_{12}) = -3$, Cofactor of $1(C_{12}) = (-1)^{1+2} \cdot M_{12} = -1 \cdot -3 = 3$

Minor of $1(M_{13}) = 1$, Co-factor of $1(C_{13}) = (-1)^{1+3} \cdot M_{13} = 1 \cdot 1 = 1$

Minor of $2(M_{21}) = -2$, Co-factor of $2(C_{21}) = (-1)^{2+1} \cdot M_{21} = -1 \cdot -2 = 2$

Minor of $1(M_{22}) = -2$, Co-factor of $1(C_{22}) = (-1)^{2+2} \cdot M_{22} = 1 \cdot -2 = -2$

Minor of $1(M_{23}) = 0$, Co-factor of $1(C_{23}) = (-1)^{2+3} \cdot M_{23} = -1 \cdot 0 = 0$

Minor of $1(M_{31}) = 0$, Cofactor of $1(C_{31}) = (-1)^{3+1} \cdot M_{31} = 1 \cdot 0 = 0$

Minor of $1(M_{32}) = -1$, Co-factor of $(C_{32}) = (-1)^{3+2} \cdot M_{32} = -1 \cdot -1 = 1$

Minor of $7(M_{33}) = -1$, Cofactor of $7(C_{33}) = (-1)^{3+3} \cdot M_{33} = 1 \cdot -1 = -1$.

Now the Co-factor matrix of $A^* = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} -2 & 3 & 1 \\ 2 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

$$Adj(A) = \begin{bmatrix} -2 & 2 & 0 \\ 3 & -2 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

Now

$$A^{-1} = \frac{Adj(A)}{|A|} = \frac{\begin{bmatrix} -2 & 2 & 0 \\ 3 & -2 & 1 \\ 1 & 0 & -1 \end{bmatrix}}{2} = \begin{bmatrix} -1 & 1 & 0 \\ 3/2 & -1 & 1/2 \\ 1/2 & 0 & -1/2 \end{bmatrix}$$

$X = A^{-1} \cdot B$.

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 3/2 & -1 & 1/2 \\ 1/2 & 0 & -1/2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2+4+0 \\ 6/2 - 4 + 1/2 \\ 3/2 + 0 - 1/2 \end{bmatrix} = \begin{bmatrix} 2+0 \\ -1+1/2 \\ 1-1/2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1/2 \\ 1/2 \end{bmatrix}$$

Comparing both the matrix.

we get, $x = 2$, $y = -1/2$, $z = 1/2$. (Ans)

(iii) $x - 2y + z = 1$ $\quad \because (|A| = 0)$ (so A^{-1} is does not exist)

$-2x + y + z = 1$

$x + y - 2z = -2$

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If $B^2 = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$, then find $B = ?$

Since the above matrix is a symmetrix.

→ Square of the matrix (B^2) is symmetric then.

B matrix also symmetric

→ then all the elements of B^2 is +ve then, all the elements of B also +ve. (According to theorem).

Sol) let $B = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$\Rightarrow B^2 = B \cdot B = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \cdot \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$\Rightarrow B^2 = \begin{bmatrix} a \cdot a + b \cdot b & a \cdot b + b \cdot a \\ b \cdot a + a \cdot b & b \cdot b + a \cdot a \end{bmatrix} = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$

$\Rightarrow B^2 = \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix} = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$

$\Rightarrow$ then equating the corresponding matrix.

$a^2 + b^2 = 17, \quad 2ab = 8.$

$a^2 + b^2 + 2ab = 17 + 8$

$\Rightarrow (a+b)^2 = 25$

$\Rightarrow a + b = 5 \quad \text{---(i)}$

$a^2 + b^2 - 2ab = 17 - 8$

$\Rightarrow (a-b)^2 = 9$

$\Rightarrow (a-b) = 9 \quad \text{---(ii)}$

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Now adding both the equation

$$a+b = 5$$
$$a-b = 3$$
$$\overline{2a = 8}$$

$\Rightarrow a = 4.$

Now substituting the value of a in eqn (1)

$$4+b = 5$$
$$\Rightarrow b = 5-4 = 1$$
$$\Rightarrow b = 1$$

Since Now $B = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ (Ans)

Q Let 'm' be a +ve integer, a, b, c are 3 real numbers. and

$$\Delta_r = \begin{vmatrix} 2r-1 & {}^{m}C_r & 1 \\ m^2-1 & 2^m & m+1 \\ a & b & c \end{vmatrix}$$

$0 \le r \le m$, then the value of $\sum_{r=0}^{m}$ is given by.

Sol

$$\sum_{r=0}^{m} = \Delta_0 + \Delta_1 + \Delta_2 + \Delta_3 - - - - - + \Delta_m$$

$$= \begin{vmatrix} -1 & {}^{m}C_1 & 1 \\ m^2-1 & 2^m & m+1 \\ a & b & c \end{vmatrix} + \begin{vmatrix} 1 & {}^{m}C_2 & 1 \\ m^2-1 & 2^m & m+1 \\ a & b & c \end{vmatrix} + \begin{vmatrix} 3 & {}^{m}C_3 & 1 \\ m^2-1 & 2^m & m+1 \\ a & b & c \end{vmatrix}$$

$$+ - - - - - - - + \begin{vmatrix} 2m-1 & {}^{m}C_m & 1 \\ m^2 & 2^m & m+1 \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} -1+1+3+5-\cdots-(2m-1) & {}^{m}C_1 + {}^{m}C_2 + {}^{m}C_3 - - - {}^{m}C_m & 1+1+1+\cdots+1 \\ m^2-1 & 2^m & m+1 \\ a & b & c \end{vmatrix}$$

$\left(\because {}^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 - - - - {}^{n}C_n = 2^n\right)$

$$= \begin{vmatrix} m^2-1 & 2^m & m+1 \\ m^2-1 & 2^m & m+1 \\ a & b & c \end{vmatrix}$$

$\left(\because S_n = \frac{n}{2}(n-1)d\right)$

$= 0$ (Ans)

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☆Properties of Transpose:-

(i) Let A & B be two matrices, then.

(i) $(A^T)^T = A$.

(ii) $(A+B)^T = A^T + B^T$

(iii) $(KA)^T = KA^T$, where K is any scalar.

(iv) $(AB)^T = B^T \cdot A^T$ [Reversal rule].

similarly $(ABC)^T = C^T \cdot B^T \cdot A^T$.

☆Properties of Adjoint:-

(i) Let A be a square matrix of order n, then

$A \cdot (adj\,A) = (adj\,A)A = |A| I_n$

where I_n is the identy matrix of order n.

eg: Give an example of 2 matrix A and B such that $AB = BA$.

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Now $adj(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

We know that $A \cdot (adj\,A) = (adj\,A)A$.

Choose $adj(A) = B$.

$\Rightarrow AB = BA$, where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(ii) $Adj(AB) = (adjB \cdot adjA)$

(iii) If A is any invertible square matrix then $adj\,A^T = (adj\,A)^T$

(iv) Let A be a non-singular ($|A| \neq 0$) square matrix of order n.

then $|adj\,A| = |A|^{n-1}$. (where n = order of the matrix)

eg: Suppose If A is a non-singular square matrix of order 3. and $|A| = 6$, then $|adj\,A| = |A|^{n-1}$

$\Rightarrow |adj\,A| = (6)^{3-1} = 6^2 = 36$.

(v) $|adj(adj\,A)| = |A|^{(n-1)^2}$

eg: If A is a non-singular square matrix of order 4. and $|A| = 6$.

then $|adj(adj\,A)| = |A|^{(n-1)^2}$

$= (6)^{(4-1)^2}$

$= 6^{(3^2)}$

$= 6^9$

(vi) $adj\,A$

(vii) $adj(adj\,A) = |A|^{n-2} \cdot A$.

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☆ Properties of Inverse:-

(i) Let A and B are invertable matrices of the same order, then $(AB)^{-1} = B^{-1} \cdot A^{-1}$ (Reversal law).

Similarly $(ABC)^{-1} = C^{-1} \cdot B^{-1} \cdot A^{-1}$.

(ii) $(A^T)^{-1} = (A^{-1})^T$.

Note:

(i) If 'A' is a matrix then $|A|$ is equal to the sum of the products of the element of any row (or any column) and the corresponding co-factors of the element.

(ii) The products of the elements of any row (or any column) and the co-factors of the elements of any other row (or column) is zero (0).

(eg-(i)) Note of (i)

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$|A| = a_{11}C_1 + a_{12}C_{12} + a_{13}C_{13}$ (Row wise)

or

$a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23}$

or

$a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$

or (Column wise)

$a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$

or

$a_{12}C_{12} + a_{22}C_{22} + a_{32}C_{32}$

or

$a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33} = |A|$

(eg-(ii)) If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

then (row wise) $a_{11} \cdot C_{21} + a_{12} \cdot C_{22} + a_{13} \cdot C_{23} = 0$.

or

$a_{21} \cdot C_{21} + a_{22} \cdot C_{32} + a_{23} \cdot C_{33} = 0$

or

$a_{21} \cdot a_{11} \cdot C_{31} + a_{12} \cdot C_{32} + a_{13} \cdot C_{33} = 0$.

or

$a_{11} \cdot C_{12} + a_{21} \cdot C_{22} + a_{31} \cdot C_{32} = 0$.

or

$a_{12} \cdot C_{13} + a_{22} \cdot C_{23} + a_{32} \cdot C_{33} = 0$.

or

$a_{11} \cdot C_{13} + a_{21} \cdot C_{23} + a_{31} \cdot C_{33} = 0$

☆ State True/False:-

(i) If a 1st row and 2nd rows of a determinant be interchanged then the sign of the determinant is changed. (T)

(ii) If 1st and 2nd rows of a determinant be interchanged then the sign of the determinant does not changed. (F)

(iii) If in a 3rd order determinant 1st row be changed to 2nd column, 2nd row to 1st column and 3rd row to 3rd column then the value of determinant is does not changed. (F).

(iv) A row and column of a determinant can have two or more common elements. (F).

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(v) The minor of the cofactor of a element a_{32} of a determinant of 3rd order equal. (F)

No 23 If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$, show that $(I+A)(I-A)^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 01 \end{bmatrix}$$

$$[I-A] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 01 \end{bmatrix}$$

$|I-A| = 1 + \tan^2\alpha/2 = \sec^2\alpha/2$

$M_{11} = 1, \quad C_{11} = 1$

$M_{12} = -\tan\alpha/2, \quad C_{12} = \tan\alpha/2$

$M_{21} = \tan\alpha/2, \quad C_{21} = -\tan\alpha/2$

$M_{22} = 1, \quad C_{22} = 1.$

Cofactor matrix of $[I-A] = \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}$

$$\text{Adj}\cdot[I-A] = \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix}$$

$$[I-A]^{-1} = \frac{\text{Adj}[I-A]}{|I-A|} = \frac{\begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix}}{\sec^2\alpha/2}$$

$$[I-A]^{-1} = \cdot \begin{bmatrix} 1/\sec^2\alpha/2 & -\tan\alpha/2/\sec^2\alpha/2 \\ \tan\alpha/2/\sec^2\alpha/2 & 1/\sec^2\alpha/2 \end{bmatrix}$$

$$[I-A]^{-1}(I+A) = \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 01 \end{bmatrix} \begin{bmatrix} \frac{1}{1+\tan^2\alpha/2} & \frac{-\tan\alpha/2}{1+\tan^2\alpha/2} \\ \frac{\tan\alpha/2}{1+\tan^2\alpha/2} & \frac{1}{1+\tan^2\alpha/2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{1+\tan^2\alpha/2} + \frac{-\tan^2\alpha/2}{1+\tan^2\alpha/2} & \frac{-\tan\alpha/2}{1+\tan^2\alpha/2} - \frac{\tan\alpha/2}{1+\tan^2\alpha/2} \\ \frac{\tan\alpha/2}{1+\tan^2\alpha/2} + \frac{\tan\alpha/2}{1+\tan^2\alpha/2} & \frac{-\tan^2\alpha/2}{1+\tan^2\alpha/2} + \frac{1}{1+\tan^2\alpha/2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2} & -\frac{2\tan\alpha/2 - \tan\alpha/2}{1+\tan^2\alpha/2} \\ \frac{\tan\alpha/2 + \tan\alpha/2}{1+\tan^2\alpha/2} & \frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & \frac{-2\tan\alpha/2}{1+\tan^2\alpha/2} \\ \frac{2\tan\alpha/2}{1+\tan^2\alpha/2} & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Proved

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Types of matrix :-

(i) **Orthogonal Matrix :=**

→ A square matrix 'A' of order n is said to be orthogonal if $AA^T = A^TA = I_n$.

(ii) **Idempotent matrix :-** A square matrix A is called idempotent matrix if A^2 i.e $A \times A = A$.

for eg → $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is idempotent.

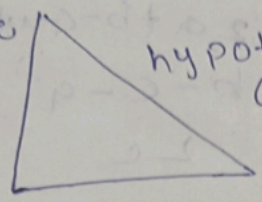
(iii) **Involuntary matrix :-** A matrix, such that $A^2 = I$ (unit matrix of the same order as A) is called an involuntary.

(iv) **Nilpotent matrix :=** A square matrix 'A' is called of nilpotent matrix if there exists a +ve integer m such that.

$$A^m = A \times A \times A \times A \cdots \text{(m times)}$$
$$= O_{m \times m}.$$

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TRIGONOMETRY



Phthagoreaces theorem

$$h^2 = p^2 + b^2$$

$$h = \sqrt{p^2 + b^2}$$

$$p^2 = h^2 - b^2$$

$$p = \sqrt{h^2 - b^2}$$

$$b^2 = h^2 - p^2$$

$$b = \sqrt{h^2 - p^2}$$

$$\frac{b}{h} = \sin\theta$$

$$\frac{b}{h} = \cos\theta$$

$$\frac{p}{b} = \tan\theta$$

$$\frac{b}{p} = \cot\theta$$

$$\frac{h}{b} = \sec\theta$$

$$\frac{h}{p} = \operatorname{cosec}\theta$$

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sec^2\theta - \tan^2\theta = 1$
3. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Proof

1. $\sin^2\theta + \cos^2\theta$

$$\left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2$$

$$\frac{p^2}{h^2} + \frac{b^2}{h^2}$$

$$\frac{p^2 + b^2}{h^2}$$

$$= \frac{h^2}{h^2} = 1$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$ $\quad \sin\theta = \sqrt{1 - \cos^2\theta}$

2. $\sec^2\theta + \tan^2\theta$

$$\left(\frac{h}{b}\right)^2 - \left(\frac{p}{b}\right)^2$$

$$= \frac{h^2}{b^2} - \frac{p^2}{b^2} = \frac{h^2 - p^2}{b^2}$$

$$= \frac{b^2}{b^2} = 1$$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec\theta = \sqrt{1 + \tan^2\theta}$

$\tan^2\theta = \sec^2\theta - 1$

$\tan\theta = \sqrt{\sec^2\theta - 1}$

③ $\operatorname{cosec}^2\theta - \cot^2\theta$

$\left(\frac{h}{b}\right)^2 - \left(\frac{p}{p}\right)^2$

$= \frac{h^2}{p^2} - \frac{h^2}{p^2} = \frac{h^2 - b^2}{p^2} = \frac{p^2}{p^2} = 1$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\boxed{\operatorname{cosec}\theta = \sqrt{1 + \cot^2\theta}}$

$\cot^2\theta =$

$\boxed{\cot\theta = \sqrt{\operatorname{cosec}^2\theta - 1}}$

④ $\sin\theta \times \operatorname{cosec}\theta = 1$

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

⑤ $\cos\theta \times \sec\theta = 1$

$\cot\theta \quad \frac{1}{\cot\theta}$

$\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$

$\cos\theta = \cot\theta \times \sin\theta$

$\cot\theta =$

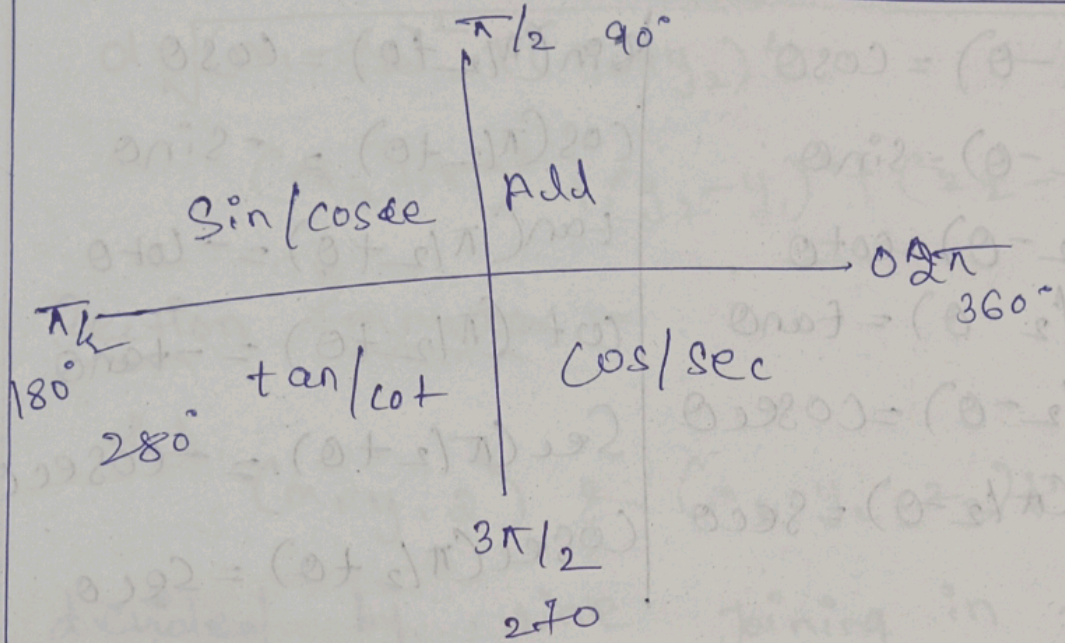
⑥ $\tan\theta \times \cot\theta = 1$

$\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$, $\frac{\cos\theta}{\sin\theta} = \cot\theta$

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\sin^2\theta}$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\sin\theta}$
$\cos\theta$	$\sqrt{1-\cos^2\theta}$	$\cos\theta$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\cos\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$
$\tan\theta$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\tan\theta$	$\frac{1}{\tan\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$
$\cot\theta$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\cot\theta}$	$\cot\theta$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sqrt{1+\cot^2\theta}$
$\sec\theta$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\sec\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sec^2\theta-1}$	$\sec\theta$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$
$\operatorname{cosec}\theta$	$\frac{1}{\operatorname{cosec}\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$	$\sqrt{\operatorname{cosec}^2\theta-1}$	$\frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$	$\operatorname{cosec}\theta$

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	0
cot	0	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	1
Cosec	0	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1



Sin (1+321) = +ve

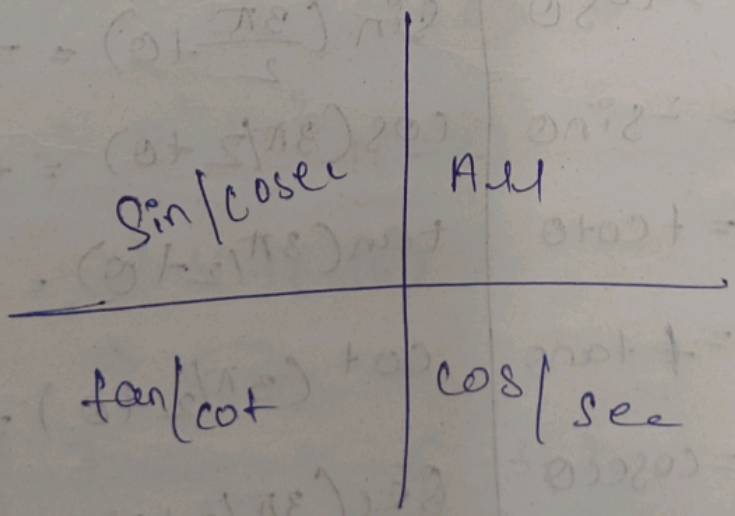
cos (3464) = -ve

tan (788) = +ve

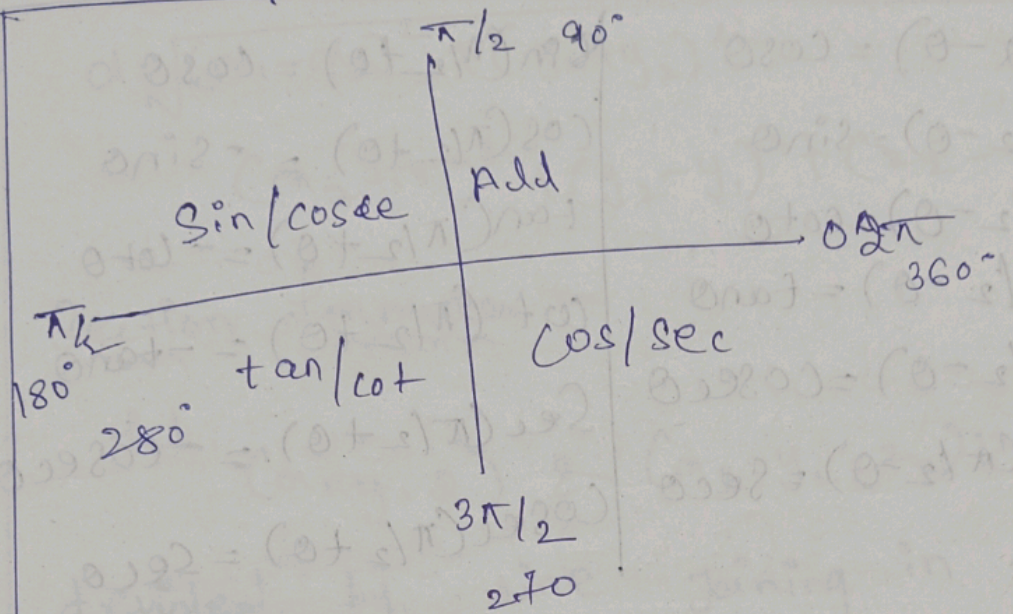
cot (921) = +ve

Sec (1882) = +ve

Cosec (6666) = -ve



A STC Rule



Sin (1+321) = +ve

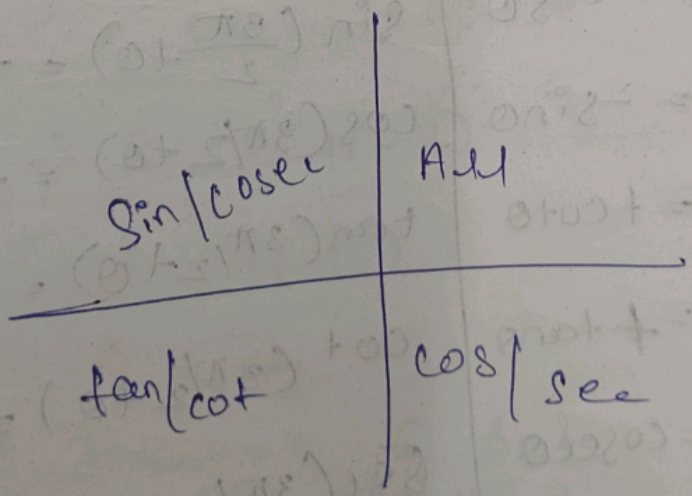
cos (3464) = -ve

tan (788) = +ve

cot (921) = +ve

sec (1882) = +ve

Cosec (6666) = -ve



$\sin(\pi/2 - \theta) = \cos\theta$

$\cos(\pi/2 - \theta) = \sin\theta$

$\tan(\pi/2 - \theta) = \cot\theta$

$\cot(\pi/2 - \theta) = \tan\theta$

$\sec(\pi/2 - \theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(\pi/2 - \theta) = \sec\theta$

$\sin(\pi/2 + \theta) = \cos\theta$

$\cos(\pi/2 + \theta) = -\sin\theta$

$\tan(\pi/2 + \theta) = -\cot\theta$

$\cot(\pi/2 + \theta) = -\tan\theta$

$\sec(\pi/2 + \theta) = -\operatorname{cosec}\theta$

$\operatorname{cosec}(\pi/2 + \theta) = \sec\theta$

$\sin(\pi - \theta) = \sin\theta$

$\cos(\pi - \theta) = -\cos\theta$

$\tan(\pi - \theta) = -\tan\theta$

$\cot(\pi - \theta) = -\cot\theta$

$\sec(\pi - \theta) = -\sec\theta$

$\operatorname{cosec}(\pi - \theta) = \operatorname{cosec}\theta$

$\sin(\pi + \theta) = -\sin\theta$

$\cos(\pi + \theta) = -\cos\theta$

$\tan(\pi + \theta) = \tan\theta$

$\cot(\pi + \theta) = -\cot\theta$

$\sec(\pi + \theta) = -\sec\theta$

$\operatorname{cosec}(\pi + \theta) = -\operatorname{cosec}\theta$

$\sin(3\pi/2 - \theta) = -\cos\theta$

$\cos(3\pi/2 - \theta) = -\sin\theta$

$\tan(3\pi/2 - \theta) = +\cot\theta$

$\cot(3\pi/2 - \theta) = +\tan\theta$

$\sec(3\pi/2 - \theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(3\pi/2 - \theta) = \sec\theta$

$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos\theta$

$\cos(3\pi/2 + \theta) = +\sin\theta$

$\tan(3\pi/2 + \theta) = -\cot\theta$

$\cot(3\pi/2 + \theta) = -\tan\theta$

$\sec(3\pi/2 + \theta) = +\operatorname{cosec}\theta$

$\operatorname{cosec}(3\pi/2 + \theta) = \ldots$

$m - 3m = 3n + 2n$

$\Rightarrow m = 3n$

$\Rightarrow m/n = 5/1 = 5:1$

Show that $\dfrac{\cos(90+\theta)\sec(-\theta)\tan(180-\theta)}{\sec(360-\theta)\sin(180+\theta)\cot(90-\theta)}$

L.H.S

$$\frac{\cos(90+\theta)\sec(-\theta)\tan(180-\theta)}{\sec(360-\theta)\sin(180+\theta)\cot(90-\theta)}$$

$$= \frac{\sin\theta \cdot \sec\theta(-\tan\theta)}{\sec\theta \cdot \sin\theta(\tan\theta)} = -1$$

$$= \frac{\sin\left(\frac{3\pi}{2}-\theta\right)\cos(\pi-\theta)}{\cos(-\theta)\sin\left(\frac{\pi}{2}+\theta\right)} = \left(\frac{-\cos\theta}{\cos\theta}\right)$$

RHS proved

- $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

  $\cos(A+B) = \cos A \cos B - \sin A \sin B$

- $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$. $\cos(A-B) = \cos A \cdot \cos B + \sin A \sin B$.

- $\tan(A+B) = \frac{\tan(A) + \tan(B)}{1 - \tan A \cdot \tan B}$ $\cot(A+B) = \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A}$

- $\tan(A-B) = \frac{\tan(A) - \tan(B)}{1 + \tan A \times \tan B}$ $\cot(A-B) = \frac{\cot A \cdot \cot B + 1}{\cot B - \cot A}$

$\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$

$\cos 15^\circ = \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \cdot \sin 30^\circ$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$

$$\tan 15^\circ = \tan(45^\circ - 30)$$

$$\frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\cot 15^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\sec 15^\circ = \frac{2\sqrt{2}}{\sqrt{3}+1}$$

$$\operatorname{cosec} 15^\circ = \frac{2\sqrt{2}}{\sqrt{3}-1}$$

$$\sin 75^\circ = \sin(15^\circ + 30^\circ)$$

$$\sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \cdot \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{3}}$$

$\cos 75^\circ = \cos(45^\circ + 30^\circ)$

$= \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \cdot \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\tan 75^\circ = \tan(45^\circ + 30^\circ)$

$\frac{\sin 75^\circ}{\cos 75^\circ} = \frac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\cot 75^\circ = \cot(45^\circ + 30^\circ)$

$= \frac{\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\sec 75^\circ \cdot \sec(45^\circ + 30^\circ)$

$= \frac{2\sqrt{2}}{\sqrt{3}+1}$

$\operatorname{cosec} 75^\circ = \operatorname{cosec}(45^\circ + 30^\circ)$

$= \frac{2\sqrt{2}}{\sqrt{2}+1}$

$\sin(A+B+C)$

$= \sin(A+(B+C))$

Let, $B+C = D$

$\sin(A+D)$

$= \sin A \cdot \cos D + \cos A \cdot \sin D$

$= \sin A \cdot \cos(B+C) + \cos A \cdot \sin(B+C)$

$= \sin A (\cos B \cdot \cos C - \sin B \cdot \sin C) + \cos A (\sin B \cdot \cos C + \cos B \sin C)$

$= \sin A \cdot \cos B \cdot \cos C - \sin A \sin B \cdot \sin C + \cos A \cdot \sin B \cdot \cos C + \cos A \cdot \cos B \cdot \sin C$

$= \sin A \cdot \cos B \cdot \cos C + \sin A \cdot \cos C + \sin C \cdot \cos A \cdot \cos B - \sin A \cdot \sin B \cdot \sin C$

$\cos(A+B+C) = \cos A \cos B \cdot \cos C - \cos A \sin B \cdot \sin C - \cos B \cdot \sin A \cdot \sin C - \cos C \cdot \sin A \cdot \sin B$

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \cdot \tan B \cdot \tan C}{1 - \tan A \cdot \tan B - \tan B \cdot \tan C - \tan A \cdot \tan C}$$

4) Prove that $\frac{\sin(A+B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cdot \cos C} + \frac{\sin(C-A)}{\cos C \cdot \cos A} = 0$

$$\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cdot \cos C} + \frac{\sin(C-A)}{\cos C - \cos A} = 0$$

$$= \frac{\sin A \cos B \cdot \cos A \sin B}{\cos A \cdot \cos B} + \frac{\sin B \cos C - \cos B \cdot \cos C}{\cos B \cos C}$$

$$+ \frac{\sin C \cdot \cos A - \cos C \cdot \sin A}{\cos C \cdot \cos A}$$

$$= \frac{\sin A \cos B}{\cos A \cdot \cos B} - \frac{\cos A \cdot \sin B}{\cos A \cdot \cos B} + \frac{\sin B \cdot \cos C}{\cos B \cdot \cos C}$$

$$\frac{\cos B \cdot \sin C}{\cos B \cdot \cos C} + \frac{\sin C \cdot \cos A}{\cos C \cdot \cos A} - \frac{\cos C \cdot \sin A}{\cos C \cdot \cos A}$$

$$\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B} + \frac{\sin B}{\cos B} - \frac{\sin C}{\cos C} + \frac{\sin C}{\cos C} - \frac{\sin A}{\cos A} = 0$$

Prove that if $A+B=45^\circ$

(1.) $(1+\tan A)(1+\tan B)=2$

$A+B=45^\circ$

$\tan(A+B)=\tan 45^\circ$

$\tan(A+B)=1$

$= \dfrac{\tan A+\tan B}{1-\tan A \times \tan B}=1$

$= \tan A+\tan B = 1-\tan A.\tan B$

$= \tan A+\tan B+\tan A.\tan B=1$

$= 1+\tan A+\tan B+\tan A.\tan B=2$

$= (1+\tan A)+\tan B(1+\tan A)=2$

$= (1+\tan A)(1+\tan B)=2$ (Proved).

(2.) Find $\tan 22\frac{1}{2}$

$22\frac{1}{2}+22\frac{1}{2}=45$

$(1+\tan 22\frac{1}{2})(1+\tan 22\frac{1}{2})=2$

$= (1+\tan 22\frac{1}{2})^2=2$

$= 1+\tan^2 22\frac{1}{2}=\sqrt{2}$

$= \tan 22\frac{1}{\sqrt{2}}=\sqrt{2}-1$

(3.) $(\cot A+1)(\cot B-1)=2$

$A+B=45^\circ$

$\cot(A+B)=\cot 45^\circ$

$$= \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A} = 1$$

$$= \cot A \cdot \cot B - 1 = \cot B + \cot A$$

$$\cot A \cdot \cot B - \cot B - \cot A = 1$$

$$\Rightarrow 1 - \cot B - \cot A + \cot A \cdot \cot B = 2$$

$$= (1 - \cot B) - \cot A (1 - \cot B) = 2$$

$$= (1 - \cot B)(1 - \cot A) = 2 \text{ (proved)}$$

- $\sin 2A = 2 \sin A \cdot \cos A$

$\sin 4A = \sin 2 \times 2A = 2 \sin 2A \cdot \cos 2A$

$\sin 6A = \sin(2 \times 3A) = 2 \sin 3A \cdot \cos 3A$

$\sin A = \sin\left(2 \times \frac{A}{2}\right) = 2 \sin \frac{A}{2} \cdot \cos \frac{A}{2}$

$\sin \frac{A}{2} = \sin\left(2 \times \frac{A}{4}\right) = 2 \sin \frac{A}{4} \cdot \cos \frac{A}{4}$

- $\cos 2A = \cos^2 A - \sin^2 A$

$\cos 4A = \cos^2 2A - \sin^2 2A$

$\cos 6A = \cos^2 3A - \tan^2 3A$

$\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}$

$\cos \frac{A}{2} = \cos^2 \frac{A}{4} - \sin^2 \frac{A}{4}$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$\tan 4A = \frac{2\tan 2A}{1-\tan^2 2A}$$

$$\tan 6A = \frac{2\tan 3A}{1-\tan^2 3A}$$

$$\tan A = \frac{2\tan \frac{A}{2}}{1-\tan^2 \frac{A}{2}}$$

$$\tan \frac{A}{2} = \frac{2\tan \frac{A}{4}}{1-\tan^2 \frac{A}{4}}$$

(1.) $\cot \frac{\pi}{8} - \tan \frac{\pi}{8} = 2$

L.H.S

$$= \frac{\cos \frac{\pi}{8}}{\sin \frac{\pi}{8}} - \frac{\sin \frac{\pi}{8}}{\cos \frac{\pi}{8}}$$

$$= \frac{\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8}}{\sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8}} = \frac{2\cos 2 \times \frac{\pi}{8}}{2\sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8}} = \frac{2\cos \frac{\pi}{4}}{\sin 2 \times \frac{\pi}{8}}$$

$$= \frac{2\cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} = \frac{2 \times \frac{1}{\sqrt{2}}}{1} = 2$$ R.H.S (proved)

(2) $\frac{\cos\theta^\circ + \sin\theta^\circ}{\cos\theta^\circ - \sin\theta^\circ} = \tan 54^\circ$

R.H.S

$\tan 54^\circ = \tan(45^\circ + \theta^\circ)$

$= \frac{\tan 45^\circ + \tan\theta^\circ}{1 - \tan 45^\circ \cdot \tan\theta^\circ} = \frac{1 + \tan\theta^\circ}{1 - \tan\theta^\circ} = \frac{1 + \frac{\sin\theta^\circ}{\cos\theta^\circ}}{1 - \frac{\sin\theta^\circ}{\cos\theta^\circ}}$

$= \frac{\frac{\cos\theta^\circ + \sin\theta^\circ}{\cos\theta^\circ}}{\frac{\cos\theta^\circ - \sin\theta^\circ}{\cos\theta^\circ}} = \frac{\cos\theta^\circ + \sin\theta^\circ}{\cos\theta^\circ - \sin\theta^\circ} = $ LHS (proved)

(3) $\tan 10^\circ + \tan 35^\circ + \tan 10^\circ \cdot \tan 35^\circ = 1$

Proof

$\tan(35^\circ + 10^\circ) = \tan 45^\circ$

$= \frac{\tan 35^\circ + \tan 10^\circ}{1 - \tan 35^\circ \cdot \tan 10^\circ} = 1$

$= \tan 35^\circ + \tan 10^\circ = 1 - \tan 35^\circ \cdot \tan 10^\circ$

$= \tan 35^\circ + \tan 10^\circ + \tan 35^\circ \cdot \tan 10^\circ = 1$ (Proved)

* The maximum value of sine is 1 & the minimum value of sine is -1
* Similarly, the maximum value of cose is and the minimum value of cose is

* Find the maximum & minimum value of $5\sin\alpha + 12\cos\alpha$.

⇒ Let $5 = r\cos\theta$, $12 = r\sin\theta$

$5^2 + 12^2 = (r\cos\theta)^2 + (r\sin\theta)^2$

$= 25 + 144 = r^2\cos^2\theta + r^2\sin^2\theta$

$= 169 = r^2(\cos^2\theta + \sin^2\theta)$

$= r^2 = 169$

$= r = 13$

$5\sin\alpha + 12\cos\alpha$

$= r\cos\theta \cdot \sin\alpha + r\sin\theta \cdot \cos\alpha$

$= r(\sin\alpha \cdot \cos\theta + \cos\alpha \cdot \sin\theta)$

$= r\sin(\alpha + \theta)$

Maximum value of $5\sin\alpha + 12\cos\alpha = r = 13$

minimum value of $(5\sin\alpha + 12\cos\alpha) = -r = -13$

Maximum value of $\sin(\alpha + \theta) = 1$

minimum value of $\sin(\alpha + \theta) = -1$

(5) $8\cos x - 15\sin x$

$\Rightarrow r\sin\theta\cdot\cos x + r\cos\theta\cdot\sin x$

$\Rightarrow r(\sin x\cdot\cos\theta + \cos x\cdot\sin\theta)$

$\Rightarrow r\sin(x+\theta)$

maximum value of $\sin(x+\theta) = 1$

minimum " " " $(x+\theta) = -1$

maximum " "

minimum " $8\cos x - 15\sin x = r = 17$

of

$8\cos x - 15\sin x = -r = -17$ (Proved)

$\cos 15^\circ = \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ\cdot\cos 30^\circ + \sin 45^\circ\cdot\sin 30^\circ$

$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\sin A\cdot\sin(60+A)\cdot\sin(60+A) = \frac{1}{4}\sin 3A$

$\cos A\cdot\cos(60-A)\cdot\cos(60+A) = \frac{1}{4}\cos 3A$

(1) $\cos 10^\circ\cdot\cos 30^\circ\cdot\cos 50^\circ\cdot\cos 70^\circ = \frac{3}{16}$

L.H.S

$\Rightarrow \cos 10^\circ\cdot\cos 30^\circ\cdot\cos 50^\circ\cdot\cos 70^\circ$

$= \cos 10^\circ\cdot(\cos 30^\circ\cdot\cos(60-10)\cdot\cos(60+10^\circ)$

$= \cos 10^\circ\cdot\cos(60-10)\cdot\cos(60+10)\cdot\cos 30^\circ$

$= \frac{1}{4}\cos(3\times 10)\cdot\cos 30^\circ$

$= \frac{1}{4}\cos 30\cdot\cos 30^\circ$

$= \frac{1}{4}\times\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{3}{16}$ R.H.S (Proved)

(2 A) LHS

$\cos 20^\circ . \cos 40^\circ . \cos 60^\circ . \cos 80^\circ$

$= \cos 20^\circ . \cos(60-20) . \cos(60+20) . \cos 60$

$= \frac{1}{4} . \cos(3\times 20) . \cos 60$

$= \frac{1}{4} . \cos 60 . \cos 60$

$= \frac{1}{4} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{16}$ R.H.S Proved

(2) (2) $\cos 20^\circ . \cos 40^\circ . \cos 60^\circ . \cos 80^\circ = \frac{1}{16}$

(roved) (3) (8) $\sin 20^\circ . \sin 40^\circ . \sin 60^\circ . \sin 80^\circ = \frac{3}{16}$

A) L.H.S

$\sin 20^\circ . \sin 40^\circ . \sin 60^\circ . \sin 80^\circ$

$= \sin 20^\circ . \sin(60-20) . \sin(60+20) . \sin 60^\circ$

$= \frac{1}{4} . \sin 3A . \sin 60^\circ$

$= \frac{1}{4} \times \sin 60^\circ . \sin 60^\circ$

$= \frac{1}{4} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} \times \frac{3}{4}$

$= \frac{3}{16}$ R.H.S (Proved)

$\Rightarrow A = 18^\circ$

$\Rightarrow 5A = 90^\circ$

$3A + 2A = 90^\circ$

$2A = 90^\circ - 3A$

$\sin 2A = \sin(90^\circ - 3A)$

$\Rightarrow \sin 2A = \cos 3A .$

$\Rightarrow 2 \sin A . \cos A = 4\cos^3 A - 3\cos A .$

$\Rightarrow 2\sin A.\cos A = 4\cos^3 A - 3\cos A$

$\Rightarrow 2\sin A.\cos A = \cos A\,(4\cos^2 A - 3)$

$\Rightarrow 2\sin A = 4\cos^2 A - 3$

$\Rightarrow 4\cos^2 A - 2\sin A - 3 = 0$

$\Rightarrow 4(1 - \sin^2 A) - 2\sin A - 3 = 0$

$\Rightarrow 4 - 4\sin^2 A - 2\sin A - 3 = 0$

$\Rightarrow -4\sin^2 A - 2\sin A + 1 = 0$

$\Rightarrow 4\sin^2 A + 2\sin A - 1 = 0$

Q) Let, $\sin A = x$

$4x^2 + 2x - 1 = 0$

$4x^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore a = 4,\ b = c,\ c = -1$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 4 \times -1}}{2 \times 4}$

$= \dfrac{-2 \pm \sqrt{4 + 16}}{8}$

$= \dfrac{-2 + \sqrt{20}}{8}$ or, $\dfrac{-2 - \sqrt{20}}{8}$

$= \dfrac{-2 + \sqrt{4 \times 5}}{8}$ or, $\dfrac{-2 - \sqrt{4 \times 5}}{8}$

$x = \dfrac{-2 + 2\sqrt{5}}{}$ or

$$x = \frac{-1+\sqrt{5}}{4} \text{ or } \frac{-1-\sqrt{5}}{4}$$

$$x = \sin A = \sin 18$$

$$= \frac{\sqrt{5}-1}{4} = \sin 18^\circ$$

Q) $\sin 8^\circ . \sin 42^\circ . \sin 60^\circ . \sin 78^\circ = \frac{1}{16}$

$$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$$

$$\cos 18^\circ = \sqrt{1-\sin^2 18^\circ}$$

$$= \sqrt{1-\left(\frac{\sqrt{5}-1}{4}\right)^2}$$

$$= \sqrt{\frac{1}{1} - \frac{(\sqrt{5}-1)^2}{16}}$$

$$= \sqrt{\frac{16-(5-2\sqrt{5}+1)}{16}}$$

$$= \sqrt{\frac{6-5+2\sqrt{5}-1}{16}}$$

$$= \sqrt{\frac{10+2\sqrt{5}}{16}}$$

$$= \sqrt{\frac{10+2\sqrt{5}}{4}} = \cos 18^\circ$$

Q) $\sin 36^\circ = \sin(2 \times 18)$

$$= 2 \times \frac{\sqrt{5}-1}{4} \cdot \sqrt{\frac{10+2\sqrt{5}}{4}}$$

$$= \frac{(\sqrt{5}-1)\left(\sqrt{10+2\sqrt{5}}\right)}{8} = \sin 36^\circ$$

1) find the maximum and minimum value of $6\cos x - 8\sin x$.

⇒ $6 = r\sin\theta$

$-8 = r\cos\theta$

$r^2\sin^2\theta + r^2\cos^2\theta = 6^2 + (-8)^2$

$\Rightarrow r^2(\sin^2\theta + \cos^2\theta) = 36 + 64$

$6\cos x - 8\sin x - 3$

$\Rightarrow r\sin\theta\cdot\cos x + r\cos\theta\cdot\sin x - 3$

$\Rightarrow r(\sin\theta\cdot\cos x + \cos\theta\cdot\sin x) - 3$

$\Rightarrow r\sin(\theta + x) - 3$

we know maximum value of $\sin(\theta + x)$ is 1 so, maximum value of $6\cos x - 8\sin x - 3$ is $r - 3 = 10 - 3 = 7$

$6 = r\sin\theta$

$-8 = r\cos\theta$

$r^2\sin^2\theta + r^2\cos^2\theta = 6^2 + (-8)^2$

$\Rightarrow r^2(\sin^2\theta + \cos^2\theta) = 36 + 64$

$6\cos x - 8\sin x - 3$

$\Rightarrow r\sin\theta\cdot\cos x + r\cos\theta\cdot\sin x - 3$

$\Rightarrow r\sin\theta\cdot\cos x + \cos\theta\cdot\sin$

we know minimum of $\sin(\theta + m)$ is -1

So,

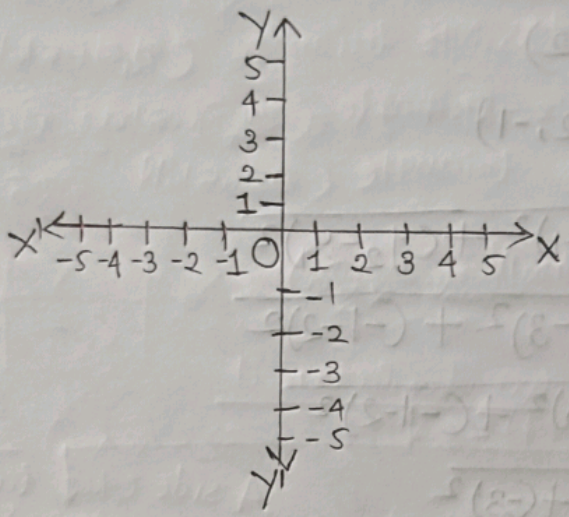
the minimum value of $6\cos m - 8\sin m - 3$ is $r - 3 = -10 - 3 = -13$

Co-ordinate geometry

Co-ordinate axis :-

*) line Y'OX and YOY' are called Co-ordinate axis.

*) X'OX and YOY' perpendicular to other and they intersect at point 'O' and it is known as origin.



→ Co-ordinate axis divides the plane in four quadrant that is XOY, X'OY, X'OY', XOY'.

1st quadrant :- $x>0, y>0$ (+,+)

2nd quadrant :- $x<0, y>0$ (−,+)

3rd quadrant :- $x<0, y<0$ (−,−)

4th quadrant :- $x>0, y<0$ (+,−)

(x, y) − Co-ordinate)

(x) → x − (Co-ordinate)

y → y − Co-ordinate

(1, 2) → 1 − x − Co-ordinate
2 − y − Co-ordinate

distance formula :-

⇒ Distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$.

$|PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Q. Find the distance between the points $P(-3, 2)$ and $Q(2, -1)$?

A⇒ $P(x_1, y_1) = (-3, 2)$

$Q(x_2, y_2) = (2, -1)$

$|PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(2-(-3))^2 + (-1-2)^2}$

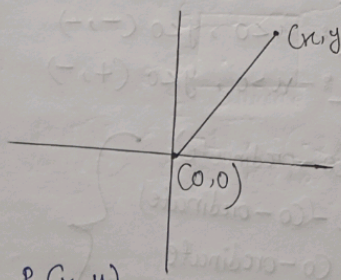
$= \sqrt{(2+3)^2 + (-1-2)^2}$

$= \sqrt{(5)^2 + (-3)^2}$

$= 25+9$

$= \sqrt{34}$ (unit)

⇒ Distance from origin to a Point $(x, y) = \sqrt{x^2+y^2}$



A⇒ O (0,0) , P (x, y)

x_1 y_1 x_2 y_2

$|OP| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(x-0)^2 + (y-0)^2}$

$= \sqrt{x^2+y^2}$

Q. if the distance between two Point $A = (3, a)$ a $B = (6, 1)$ is 5, find the value of a ?

A⇒ $A(x_1, y_1) = (3, a)$

$B(x_2, y_2) = (6, 1)$

$|AB| = 5$

$|AB| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= 5^2 = \left(\sqrt{(6-3)^2 + (1-a)^2}\right)^2$

$= 5^2 = 3^2 + (1-a)^2$

$= (1-a)^2 = 5^2 - 3^2$

$= (1-a)^2 = 25 - 9 = 16$

$= (1-a)^2 = 16$

$= 1-a = \sqrt{16} = \pm 4$

$= -a = \pm 4 - 1 = 4-1$ or $-4-1$

$= -a = 3$ or -5

$= a = -3$ or 5

Triangle

Triangle (ତ୍ରିଭୁଜ) :-

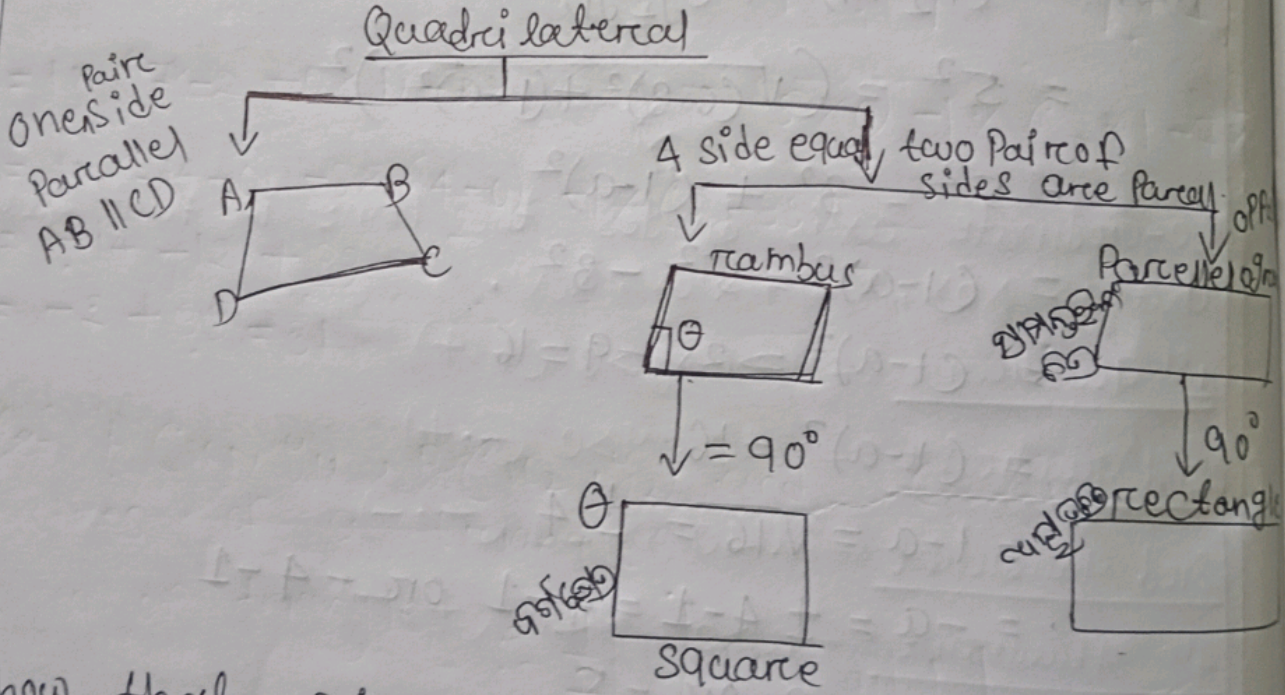
→ According to the side :- ('3' types).

(i) Scalene triangle (ବିଷମବାହୁ) –
(ii) Isoscele's triangle (ସମଦ୍ୱିବାହୁ) –
(iii) Equilateral triangle (ସମବାହୁ) –

→ According to the angle :- ('3' types).

(i) A cute ~~trai~~angle triangle (ସୂକ୍ଷ୍ମକୋଣୀ ତ୍ରିଭୁଜ) = θ <
(ii) Abtuse angle triangle (ସ୍ଥୂଳକୋଣୀ ତ୍ରିଭୁଜ) = -θ >
(iii) Right angle triangle (ସମକୋଣୀ ତ୍ରିଭୁଜ) = one angle = 90°

(Quadrilateral) means (- ଚତୁର୍ଭୁଜ)



Q. Show that Points (0, -1), (-2, 3), (6, 7), and (8, 3) are vertices of a rectangle ?

Ans)

Determinate

Q) $\begin{vmatrix} 1 & 2 & 5 \\ 3 & 7 & 1 \\ -1 & 0 & -2 \end{vmatrix} = 1 \times \begin{vmatrix} 7 & 1 \\ 0 & -2 \end{vmatrix} - 2 \times \begin{vmatrix} 3 & 1 \\ -1 & -2 \end{vmatrix} + 5 \times \begin{vmatrix} 3 & 7 \\ -1 & 0 \end{vmatrix}$

$= 1 \times (7 \times -2 - 0 \times 1) - 2 \times (3 \times -2 - 1 \times -1) + 5 \times (3 \times 0 - 7 \times -1)$

$= 1 \times (-14 - 0) - 2 \times (-6 + 1) + 5 (0 + 7)$

$= -14 - 2 \times -5 + 5 \times 7$

$= -14 + 10 + 35 = -14 + 45 = 31$

2) $\begin{vmatrix} + & - & + \\ 1 & 2 & 5 \\ 3 & 7 & 1 \\ -1 & 0 & -2 \end{vmatrix} = 1 \times \begin{vmatrix} 7 & 1 \\ 0 & -2 \end{vmatrix} - 3 \times \begin{vmatrix} 2 & 5 \\ 0 & -2 \end{vmatrix} - 1 \times \begin{vmatrix} 2 & 5 \\ 7 & 1 \end{vmatrix}$

$= 1 \times (7 \times -2 - 0 \times 1) - 3 \times (2 \times -2 - 5 \times 0) - 1 \times (2 \times 1 - 7 \times 5)$

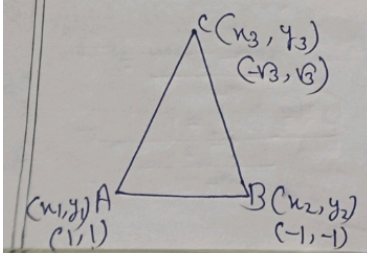
$= 1 \times (-14 - 0) - 3 \times (4 - 0) - 1 \times (2 - 35)$

$= -14 - 12 + 33$

$= -2 + 33 = 31$

Q) Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of a triangle?

A) area of triangle $= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$



Area of $\Delta = \frac{1}{2} \times \begin{vmatrix} 1 & 1 & 1 \\ -1 & -1 & 1 \\ -\sqrt{3} & \sqrt{3} & 1 \end{vmatrix}$

$= \frac{1}{2} \left\{ 1 \begin{vmatrix} -1 & 1 \\ \sqrt{3} & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 1 \\ \sqrt{3} & 1 \end{vmatrix} - \sqrt{3} \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} \right\}$

$= \frac{1}{2} (-1 - \sqrt{3}) + (1 + \sqrt{3}) - \sqrt{3} (1 + 1)$

$= \frac{1}{2} \times -2\sqrt{3} - -\sqrt{3}$

$\therefore$ Area of $\Delta = \sqrt{3}$ square unit.

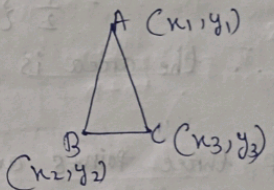
Proof

→ Let ABC be a triangle

A (x_1, y_1)

B (x_2, y_2)

C (x_3, y_3)



Then area of $\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Q) find the area of Δ if three vertices are (3,1), (3,4), (6,4)?

A) $(x_1, y_1) = (3,1)$ $(x_3, y_3) = (6,4)$

$(x_2, y_2) = (3,4)$

area of $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 3 & 4 & 1 \\ 6 & 4 & 1 \end{vmatrix}$

$= \frac{1}{2}\left\{ 3\times\begin{vmatrix} 4 & 1 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} + 6\times\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} \right\}$

$= \frac{1}{2}\{3\times(4-4) - 3(1-4) + 6(1-4)\}$

$= \frac{1}{2}\{3\times 0 - 3\times -3 + 6\times 3\}$

$= \frac{1}{2}\{9-18\} = \frac{-9}{2}$

∴ the area is the $\frac{9}{2}$ cenit.

If three points are colinear than

Colinear formula :- $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

|AB| + |BC| = |AC|

A B C

Q) find the value of a so that points (1,4), (2,7), (3,a) are colinear?

A) $(x_1, y_1) = (1,4)$

$(x_2, y_2) = (2,7)$

$(x_3, y_3) = (3,a)$

$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 2 & 7 & 1 \\ 3 & a & 1 \end{vmatrix} = 0$

$\Rightarrow 1\times\begin{vmatrix} 7 & 1 \\ a & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 1 \\ a & 1 \end{vmatrix} + 3\begin{vmatrix} 4 & 1 \\ 7 & 1 \end{vmatrix} = 0$

$\Rightarrow (7-a) - 2(4-a) + 3(4-7) = 0$

$\Rightarrow 7 - a - 8 + 2a - 9 = 0$

$\Rightarrow a - 10 = 0$

$\Rightarrow a = 10$

Q) If the area of Δ with vertices (0,0), (1,0), (0,a) is 10 find the value of a?

A) area of triangle $= \frac{1}{2}\begin{vmatrix} +0 & 0 & 1 \\ -1 & 0 & 1 \\ +0 & a & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 0 \\ 0 & a \end{vmatrix} = \frac{1}{2}$

$(a-0) = \frac{a}{2}$

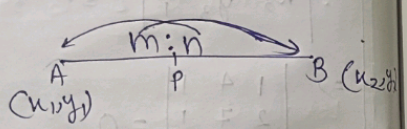
$= \frac{1}{2}(-1)\begin{vmatrix} 0 & 1 \\ a & 1 \end{vmatrix}$

$= \frac{-1}{2}(0-a)$

$= \frac{-1}{2} \times (-a) = \frac{a}{2}$

$\therefore \frac{a}{2} = 10 \Rightarrow a = 10 \times 2 = 20$ (unit)

Internal divison :-



$\frac{AP}{PB} = \frac{m}{n}$

$P(\bar{x}, \bar{y})$

comp

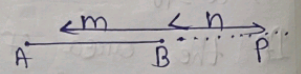
$\bar{x} = \frac{mx_2 + nx_1}{m+n}$

$\bar{y} = \frac{my_2 + ny_1}{m+n}$

comp **mid Point**

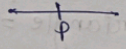
$\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

External divison :-



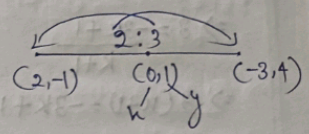
$\bar{x} = \frac{mx_2 - nx_1}{m-n}$

$\bar{y} = \frac{my_2 - ny_1}{m-n}$



Q) Find the co-ordinates of the points which divides PQ in the ratio 2:3 internally and externally P(2, -1), Q(-3, 4) ?

A) **internal divison**



$\bar{x} = \frac{2 \times (-3) + 3 \times 2}{2+3}$

$= \frac{-6+6}{5} = \frac{0}{5} = 0$

$\therefore x = 0, y = 1$

$\bar{y} = \frac{2 \times 4 + 3 \times (-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$

External division

$\bar{x} = \frac{2 \times (-3) - 3 \times 2}{2-3}$

$= \frac{-6-6}{-1} = \frac{-12}{-1} = 12$

$\bar{y} = \frac{2 \times 4 - 3 \times (-1)}{2-3}$

$= \frac{8+3}{-1} = \frac{11}{-1} = -11$

$\therefore x = 12, y = -11$

Q3) in what ratio does the points $(3,-2)$ divide the line segment joining the points $(1,4)$, $(-3,16)$?

A3) $3 = \frac{k \times (-3) + 1 \times 1}{k+1}$

$\Rightarrow 3 = \frac{-3k+1}{k+1}$

$\Rightarrow 3(k+1) = -3k+1$

$\Rightarrow 3k+3 = -3k+1$

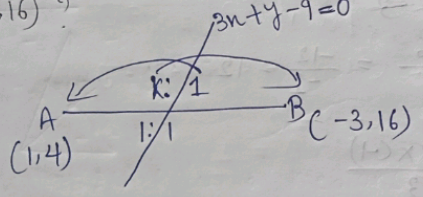
$\Rightarrow 3k+3k = 1-3 = -2$

$\Rightarrow 6k = -2$

$\Rightarrow k = \frac{-2}{6} = \frac{-1}{3}$

Q3) find the ratio in which the line $3x+y-9=0$ divides the line segment of the point $(1,4)$, and $(-3,16)$?

A3)



$\left(\frac{k \times (-3) + 1 \times 1}{k+1}, \frac{k \times 16 + 1 \times 4}{k+1}\right)$

$= \left(\frac{-3k+1}{k+1}, \frac{16k+4}{k+1}\right)$

$3x+y-9=0$

$\Rightarrow 3x\left(\frac{-3k+1}{k+1}\right) + \frac{16k+4}{k+1} = 9$

$\Rightarrow \frac{-9k+3}{k+1} + \frac{16k+4}{k+1} = 9$

$\Rightarrow \frac{-9k+3+16k+4}{k+1} = 9$

$\Rightarrow \frac{7k+7}{7k+7} = 9 \quad (k+1)$

$\Rightarrow 7k+7 = 9k+9$

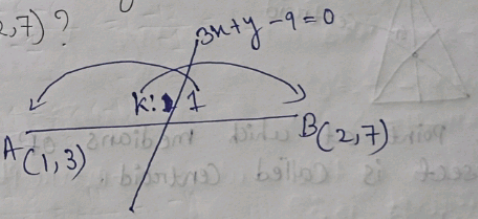
$\Rightarrow 7k-9k = 9-7$

$\Rightarrow -2k = 2$

$\Rightarrow k = 1$

Q4) find the ratio in which the line $3x+y-9=0$ divides the line segment of the Point $(1,3)$, and $(2,7)$?

A4)



$\left(\frac{k \times 2 + 1 \times 1}{k+1}, \frac{k \times 7 + 1 \times 3}{k+1}\right)$

$= \left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$

$= 3x + y - 9 = 0$

$\Rightarrow 3 \times \left(\frac{2k+1}{k+1}\right) + \frac{7k+3}{k+1} = 9$

$\Rightarrow \frac{6k+3}{k+1} + \frac{7k+3}{k+1} = 9$

$\Rightarrow \frac{6x+7x+3+3}{k+1} = 9k+9$

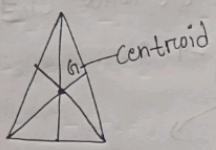
$\Rightarrow 13k + 6 = 9k + 9$

$\Rightarrow 13k - 9k = 9 - 6$

$\Rightarrow 4k = 3$

$\Rightarrow k = \frac{3}{4}$

Centroid :-



→ the point at which medians of triangle interseect is called centroid.

Centroid formula :- $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Q) Two vertices of triangle are $(0, -4)$ and $(6$ if the medians meet at the point $(2,0)$ fin the co-ordinates of the 3rd vertiex ?

A) **Centroid**

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$= \left(\frac{0+6+x_3}{3}, \frac{-4+0+y_3}{3}\right)$

$= \frac{6+x_3}{3}, \frac{-4+y_3}{3} = (2,0)$

$= \frac{6+x_3}{3} = 2$

$= 6 + x_3 = 6$

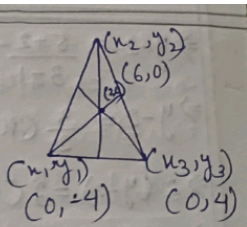
$= x_3 = 0$

$\frac{-4+y_3}{3} = 0$

$\Rightarrow -4 + y_3 = 0$

$\Rightarrow y_3 = 4$

$\therefore (x_3 = 0, y_3 = 4)$



Q) find the equation of line passes through $(1,$ and $(3,5)$?

A) $(x_1, y_1) = (1,2)$

$(x_2, y_2) = (3,5)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y-2 = \frac{5-2}{3-1}(x-1)$

$\Rightarrow y-2 = \frac{3}{2}(x-1)$

$\Rightarrow 2(y-2) = 3(x-1)$

$\Rightarrow 2y-4 = 3x-3$

$\Rightarrow 3x-2y = -4+3$

$\Rightarrow 3x-2y = -1$

Q2) find the equation of line Passes through (1,2) and slope −2?

A) $(x_1, y_1) = (1,2)$

$m = -2$

equation of line

$y-y_1 = m(x-x_1)$

$\Rightarrow y-2 = (-2)\cdot(x-1)$

$\Rightarrow y-2 = -2x+2$

$\Rightarrow 2x+y-2-2=0$

$\Rightarrow 2x+y-4=0$

Q3) equation of line, whose x-intercept is −2 and y intercept is 3?

A) $a = -2$

$b = 3$

equation of line

$\frac{x}{a} + \frac{y}{b} = 1$ (x, y)

$\Rightarrow \frac{-x}{2} + \frac{y}{3} = 1$

$\Rightarrow \frac{-3x+2y}{6} = 1$

$\Rightarrow -3x+2y = 6$

$\Rightarrow 3x-2y = -6$

Q4) find the equation of line passes through Point (−2, 3) and sum of whose intercept is 2?

A) $a+b = 2$

$\Rightarrow b = 2-a$

$\frac{x}{a} + \frac{y}{b} = 1$

$= \frac{x}{a} + \frac{y}{2-a} = 1$

$\Rightarrow \frac{-2}{a} + \frac{3}{2-a} = 1$

$\Rightarrow \frac{-2\times 1 - a + 3a}{a(2-a)} = 1$

$\Rightarrow \frac{-4+2a+3a}{a-a^2} = 1$

$\Rightarrow -4+5a = a-a^2$

$\Rightarrow a^2+5a-a-2=0$

$\Rightarrow a^2+3a-4=0$

$\Rightarrow a^2 - a + 4a - 4 = 0$

$\Rightarrow a(a-1) + 4(a-1) = 0$

$\Rightarrow a-1=0$ or $a+4=0$

$\Rightarrow a=1$ or $a=-4$

equation of line

$\frac{x}{1} + \frac{y}{2-1} = 1$

$\Rightarrow x + y = 1$

$\Rightarrow \frac{x}{-4} + \frac{y}{6} = 1$

$\Rightarrow \frac{-x+4y}{24} = 1$

$\Rightarrow -6x + 4y = 24$

$\Rightarrow 6x - 4y = 24$

Q) find the equation of line passes through Point $(-2,3)$ and make angle 30° with +ve directing X axis?

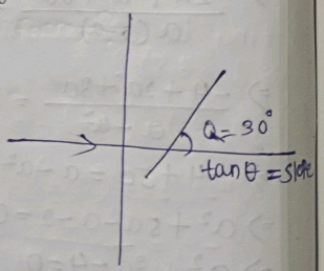
A) $y - y_1 = m(x - x_1)$

$(x_1, y_1) = (-2, 3)$

slope $= m = \tan 30^\circ = \frac{1}{\sqrt{3}}$

equation of line

$y - 3 = \frac{1}{\sqrt{3}}(x_1 - (-2))$



$\Rightarrow \sqrt{3}(y-3) = x+2$

$\Rightarrow x - \sqrt{3}y + 3\sqrt{3} + 2 = 0$

formula:-

Slope – intercept form :-

$y = mx + C$ → y-intercept

↓ slope

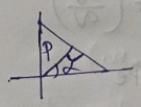
intercept form :-

$\frac{x}{a} + \frac{y}{b} = 1$ → y-intercept

↓ x-intercept

Normal form :-

$x\cos\alpha + y\sin\alpha = p$



slope formula :-

i) $m = \tan\theta$

ii) $m = \frac{y_2 - y_1}{x_2 - x_1}$

iii) $ax + by + C = 0$

$\Rightarrow by = -ax - C$

$\Rightarrow y = \frac{-a}{b}x - \frac{c}{b}$

passing through the two points :-

$= y - y_1$

$= \frac{y_2 - y_1}{x_2 - x_1}$

Value :-

$\cos\alpha = \frac{-p}{r}$ $\quad p = \frac{-R}{\sqrt{p^2+q^2}}$

$\sin\alpha = \frac{-q}{r}$ $\quad p = \frac{-q}{\sqrt{p^2+q^2}}$

$p = \frac{\pm r}{\sqrt{p^2+q^2}}$

General equation of line :- (Convert intercept form)

$px+qy+r=0$

$y = mx+c$

$px+qy+r=0$

$\Rightarrow qy = -px-r$

$\Rightarrow y = \left(\frac{-p}{q}\right)x + \left(\frac{-r}{q}\right)$

$m = \frac{-p}{q}$ – slope

$c = \frac{-r}{q}$ – intercept

Q5) $4x+7y=12$ (Slope intercept form) ?

A) $y = mx+c$

$\Rightarrow 7y = -4x+12$

$\Rightarrow y = \frac{-4}{7}x + \frac{12}{7}$

$\Rightarrow m = \frac{-4}{7}$ $\quad c = \frac{12}{7}$

Q2) $2x+3y=5$ (intercept form) ?

A) $\frac{2x}{5} + \frac{3y}{5} = \frac{5}{5}$

$\Rightarrow \frac{2}{5}x + \frac{3}{5}y = 1$

$\Rightarrow \frac{\left(\frac{2}{5}\right)}{\left(\frac{5}{2}\right)}x + \frac{\left(\frac{3}{5}\right)}{\left(\frac{5}{3}\right)}y = 1$

$\Rightarrow \frac{x}{\left(\frac{5}{2}\right)} + \frac{y}{\left(\frac{5}{3}\right)} = 1$

$\Rightarrow a = \frac{5}{2}$, $b = \frac{5}{3}$

Q3) $2x+3y=5$ (intercept form) ?

A) $\frac{2x}{5} + \frac{3y}{5} = 1$

$\Rightarrow \frac{x}{\left(\frac{5}{2}\right)} + \frac{y}{\left(\frac{5}{3}\right)} = 1$

Q4) $2x-3y+4=0$

Slope intercept form ?

A) 1st method

$y = mx+c$

$\Rightarrow \frac{-3y}{-3} = \frac{-2x}{-3} \quad \frac{-4}{-3}$

$\Rightarrow y = \frac{2}{3}x + \frac{4}{3}$

2nd method

$\frac{2x}{3} + \frac{4}{3} = \frac{3y}{3}$

$\Rightarrow y = \frac{2}{3}x + \frac{4}{3}$

Q5) $ax+by+c=0$ (normal form)
$2x+3y-7=0$

A:) $x\cos\alpha + y\sin\alpha = p$

$\cos\alpha = \frac{-2}{\sqrt{2^2+3^2}}$ $\quad p = \frac{\pm 7}{\sqrt{2^2+3^2}}$

$\sin\alpha = \frac{-3}{\sqrt{2^2+3^2}}$

Q6) $2x+3y-7=0$ (Direct form)

$\Rightarrow -2x-3y+7=0$

$\Rightarrow \frac{-2x}{\sqrt{2^2+3^2}} - \frac{3y}{\sqrt{2^2+3^2}} = \frac{\pm 7}{\sqrt{2^2+3^2}}$

(formula)

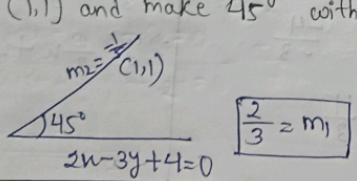
$y-y_1 = m(x-x_1)$

equation of line passes through a point and slope $-m$

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

Q1) Find the equation of line passes through the point $(1,1)$ and make 45° with line $2x-3y+4=0$?

Ans)



$\boxed{\frac{2}{3} = m_1}$

$\tan\theta = \frac{m_1-m_2}{1+m_1 \cdot m_2}$

$\Rightarrow \tan 45^\circ = \frac{\frac{2}{3}-m_2}{1+\frac{2}{3}\cdot m_2}$

$\Rightarrow 1 = \frac{\frac{2}{3}-m_2}{1+\frac{2m_2}{3}}$

$\Rightarrow \frac{1+2m_2}{3} = \frac{2}{3} - m_2$

$\Rightarrow \frac{2m_2}{3} + m_2 = \frac{2}{3} - 1$

$\Rightarrow \left(\frac{2+3}{3}\right) + m_2 = \frac{2-3}{3}$

$\Rightarrow \frac{5}{3} m_2 = \frac{-1}{3}$

$\Rightarrow m_2 = \left(\frac{-1}{5}\right)$

$y-y_1 = m(x-x_1)$

$\Rightarrow y-1 = \frac{-1}{5}(x-1)$

$\Rightarrow 5y-5 = -x+1$

$\Rightarrow x+5y-6=0$

formula (Slope)

Parallel :- $\Rightarrow m_1 = m_2$

Perpendicular:-

$m_1 \cdot m_2 = (-1)$

Angle $= \theta$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

general equation:-

$a_1 x + b_1 y + c_1 = 0$

$a_2 x + b_2 y + c_2 = 0$

Parallel:-

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Coincident

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Perpendicular:-

$a_1 a_2 + b_1 b_2 = 0$

→ distance between two Parallel line

$a_1 x + b_1 y + c_1 = 0$

d) $a_1 x + b_1 y + c_2 = 0$

$d = \frac{|d_1 - d_2|}{\sqrt{a_1^2 + b^2}}$

Q1) $3x + ky - 4 = 0$ (coincident k)

$k - 4y - 3x = 0$

A) $a_1 = 3$, $b_1 = k$, $c_1 = -4$

$a_2 = -3$, $b_2 = -4$, $c_2 = k$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{-3}{-3} = \frac{k}{-4} = \frac{-4}{k} = -1$

$\Rightarrow \frac{k}{-4} = -1$

$\Rightarrow k = -1 \times -4 = 4$

Q2) $x - 2y + 2 = 0$

$2x + ky - 1 = 0$ have same y intercept ?

A) $x - 2y = -2$

$\Rightarrow \frac{x}{-2} - \frac{2y}{-2} = 1$

$\Rightarrow \frac{x}{-2} + \frac{y}{1} = 1$

$\Rightarrow 2x + ky = 1$

$\Rightarrow \frac{x}{(\frac{1}{2})} + \frac{y}{(\frac{1}{k})} = 1$

$\Rightarrow k = 1$

Q3) $2x - 3y + 4 = 0$

$4x + ky + 1 = 0$ are Parallel then find the valu

A) $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

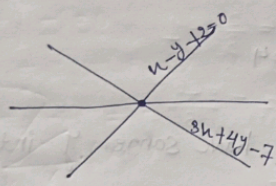
$\Rightarrow \frac{2}{4} = \frac{-3}{k}$

$\Rightarrow 2 \times k = -3 \times 4$

$k = \frac{-3 \times 4}{2} = -6$

Q4) Find the equation of line passing through point of intersection of line $3x+4y-7=0$ and $x-y+2=0$ which is parallel to line $5x-y+11=0$?

A)



$5x-y+11=0$

$\Rightarrow 3x+4y-7=0$

$8x-3y+6=0$

$7y-13=0$

$y = \frac{13}{7}$

$x - \frac{13}{7} + 2 = 0$

$\Rightarrow x = \frac{13}{7} - 2 = \frac{13-14}{7} = \frac{-1}{7}$

m (1,1)

$(x_1, y_1) = \left(\frac{-1}{7}, \frac{13}{7}\right)$

$m = 5$

$y_1 - y_1 = m(x - x_1)$

$\Rightarrow y - \frac{13}{7} = 5\left(x - \frac{(-1)}{7}\right)$

$\frac{7y-13}{7} = \frac{5(7x+1)}{7}$

$\Rightarrow 35x + 5 - 7y + 13 = 0$

$\Rightarrow 35x - 7y + 18 = 0$

formula

line equation slope - in point $= (x_1, y_1)$

m (a,b)

$y - y_1 = m(x - x_1)$

Q5) Find the equation of line passing through $(-4, 2)$ and inclined at an angle 45° with the x-axis is?

A) Slope $= m = \tan 45^\circ = 1$

Point $= (x_1, y_1) = (-4, 2)$

$y - y_1 = m(x - x_1)$

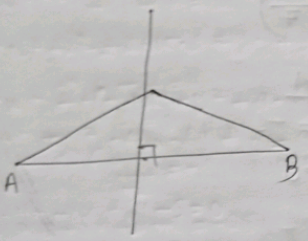
$\Rightarrow y - 2 = 1(x - -4)$

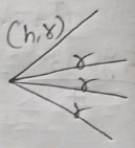
$\Rightarrow y - 2 = x + 4$

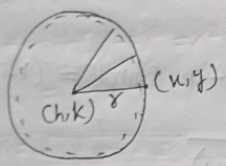
$\Rightarrow y - 2 - x - 4 = 0 \Rightarrow -x + y - 6 = 0 \Rightarrow x - y + 6 = 0$

Circle :-

⇒ A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point is constant.







$= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} = r$

$= \sqrt{(x-h)^2+(y-k)^2} = r$

$= (x-h)^2+(y-k)^2 = r^2$

Equation of circle.

Centre $= (h,k)$

Radius $= r$

Q) centre $= (1,2) = (h,k)$

Radius $= 3 = r$

Ans) $(x-h)^2+(y-k)^2 = r^2$

$\Rightarrow (x-1)^2+(y-2)^2 = 3^2$

$\Rightarrow x^2+1^2-2\cdot x\cdot 1+y^2+2^2-2\cdot y$

$\Rightarrow x^2-2x+y^2-4y+1+4=9$

General equation of circle :-

(Formula)

$(x^2+y^2+2gx+2fy+c=0)$

Centre

$(h,k) = (-g,-f)$

$(r = \sqrt{g^2+f^2-c})$

↓ radius

Q) equation of circle Radius $= r$
Centre $= (h,k)$

equation of circle ?

Ans) $(x-h)^2+(y-k)^2 = r^2$

$x^2+y^2+2gk+2fy+c=0$

centre $(h,k) = (-g,-f)$

radius $= r = \sqrt{g^2+f^2-c}$

let, Radius $= 3$cm

centre = (-2, 3)

The equation circle

$=(x-(-2))^2 + (y-3)^2 = 3^2$

$\Rightarrow (x+2)^2 + (y-3)^2 = 3^2$

$\Rightarrow x^2 + 2^2 + 2 \cdot x \cdot 2 + y^2 + 3^2 - 2 \cdot y \cdot 3 = 9$

$\Rightarrow x^2 + 4 + 4x + y^2 + 9 - 6y = 9$

$\Rightarrow x^2 + y^2 + 4x - 6y + 13 = 9$

Q2) $x^2 + y^2 - 8x + 6y - 5 = 0$

A2) $(x^2 + y^2 + 2gx + 2fy + C = 0)$ → general equation of circle

$(x-h)^2 + (y-k)^2 = r^2$

$(h, k) = (-g, -f)$

$r = \sqrt{g^2 + f^2 - C}$

$2g = -8 \Rightarrow g = \frac{-8}{2} = -4$

$2f = 6 \Rightarrow f = \frac{6}{2} = 3$

$C = -5$

centre $(h, k) = (-g, -f) = (-(-4), -3)$

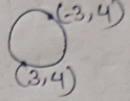
$= (4, -3)$

Radius $r = \sqrt{g^2 + f^2 - C}$

$= \sqrt{(-4)^2 + 3^2 - (-5)} = \sqrt{16 + 9 + 5} = \sqrt{30}$

Q3) Find the equation of circle passing through the points (3, 4), (4, -5) and (-3, 4) ?

A3)



general equation of circle

$x^2 + y^2 + 2gx + 2fy + C = 0$

(4, -5) (-3, 4)

(x_2, y_2)

$3^2 + 4^2 + 2 \times g \times 3 + 2f \times 4 + C = 0$

$\Rightarrow 9 + 16 + 6g + 8f + C = 0$

$\Rightarrow 6g + 8f + C = -25$ —— (1)

$4^2 + (-5)^2 + 2 \times g \times 4 + 2 \times f \times (-5) + C = 0$

$\Rightarrow 16 + 25 + 8g - 10f + C = 0$

$\Rightarrow 8g - 10f + C = -41$ —— (2)

$(-3)^2 + 4^2 + 2 \times g \times (-3) + 2 \times f \times 4 + C = 0$

$\Rightarrow 9 + 16 - 6g + 8f + C = 0$

$\Rightarrow -6g + 8f + C = -25$ —— (3)

$6g + 8f + C = -25$ —— (1)

$8g + 10f + C = -41$ —— (2)

equation (3) − equation (1)

$-6g + 8f + C = -25$

$\underline{6g + 8f + C = -25}$

$\Rightarrow -12g = 0 \Rightarrow g = 0$

equation ① − equation ②

$6g+8f+C=-25$

$8g+10f+C=-41$

$-2g-2f=16$

$-2\times0-2f=16$

$\Rightarrow 0-2f=16$

$\Rightarrow -2f=16$

$\Rightarrow f=\frac{16}{-2}=-8$

$-6\times0+8\times-8+C=-25$

$\Rightarrow -64+C=-25$

$\Rightarrow C=-25+64=39$

Equation of circle

$\boxed{x^2+y^2-16y+39=0}$

Q4) Find the equation of circle passing through the points $(1,3)$, $(2,-1)$ and $(-1,1)$?

A) general equation of circle

$\boxed{x^2+y^2+2gx+2fy+C=0}$

$(x=1,\ x=2,\ x=-1)$

$(y=3,\ y=-1,\ y=1)$

$1^2+3^2+2\times g\times1+2f\times3+C=0$

$\Rightarrow 1+9+2g+6f+C=0$

$\Rightarrow 2g+6f+C=-10$ — ①

$\Rightarrow 2^2+-1^2+2\times g\times2+2f\times-1+C=0$

$\Rightarrow 4+(+1)+4g+(-2)f+C=0$

$\Rightarrow 4g+(-2)f+C=-5$ — ②

$\Rightarrow (-1)^2+1^2+2\times g\times(-1)+2f\times1+C=0$

$\Rightarrow 1+1+-2g+2f+C=0$

$\Rightarrow -2g+2f+C=-2$ — ③

eqn ① − eqn ②

$2g+6f+C=-10$

$4g+(-2f)+C=-5$

$-2g+8f=-5$

eqn ③ − eqn ①

$-2g+2f+C=-2$

$2g+6f+C=-10$

$-4g-4f=8$

$\Rightarrow -4(g+f)=8$

$\Rightarrow g+f=\frac{8}{-4}=-2$

$-2g+8f=-5$

$2g+2f=-4$

$10f=-9$

$f=\frac{-9}{10}$

$g+\frac{-9}{10}=-2$

$\Rightarrow g=-2+\frac{9}{10}=\frac{-20+9}{10}=\frac{-11}{10}$

$-2g+2f+C=-2$

$\Rightarrow -2\times\frac{-11}{10}+2\times\frac{-9}{10}+C=-2$

$\Rightarrow \frac{+22}{10}-\frac{18}{10}+C=-2$

$\Rightarrow \frac{22-18}{10}+C=-2$

$\Rightarrow \frac{4}{10}+C=-2$

$\Rightarrow C=-2-\frac{4}{10}^{2}$

$\Rightarrow -2-\frac{2}{5}$

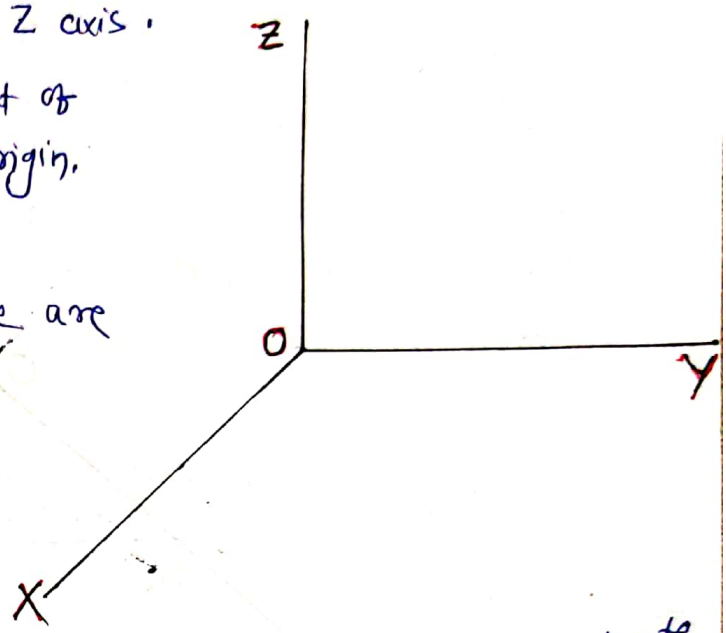
$\Rightarrow \frac{-10-2}{5}=\frac{-12}{5}$

THREE - DIMENSION (3-D)

→ There are three lines which are mutually perpendicular to each other Viz. X axis, Y axis, Z axis.

'O' is the intersepticpoint of X axis, Y axis & Z axis. Called as origin.

O(0,0,0), P(x,y,z).



→ In ~~3~~ to 3-D (three dimension) there are 3 planes. Viz. XY-plane
YZ - plane
ZX - plane.

→ Any point in 3-D, there are three co-ordinates i.e x-co-ordinate, y-co-ordinate, Z-coordinate.

eg → P(2,3,4).

Here '2' is the x-coordinate
'3' is the y-co-ordinate.
'4' is the Z-Co-ordinate.

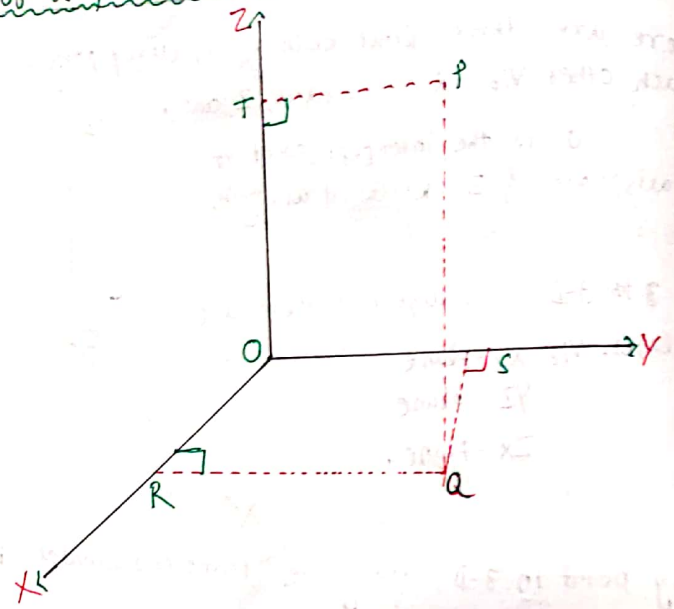
→ In 3-D there are 8 spaces i.e also called as octant.

(+,+,+), (+ + −), (+ − +) (+ − −)
(− + +), (− + −), (− − +), (− − −)

x	y	z
+	+	+
+	+	−
+	−	+
+	−	−
−	+	+
−	+	−
−	−	+
−	−	−

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☆ Position of a point on the Space:-



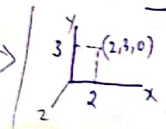
Let $P(x,y,z)$ be any point on the Space.
Draw the perpendicular from the 'P' to XY plane meet at Q. Draw the perpendicular QR on x-axis. and QS on Y-axis. and PT on Z-axis.

Here OR is x unit
OS is y unit.
& OT is z unit

$OR = SQ = x$ unit
$OS = RQ = y$ unit. } ∵ ORSQ is a rectangle.
$OT = PQ = z$ unit. (∵ lie in one plane i.e XY-plane)

→ Any point on XY-plane whose Z-coordinate is Zero (0).
So, any point lie the XY plane in the form of $P(x,y,0)$.

e.g → $P(2,3,0)$ →



Similarly any point lies on yz-plane in the form of $P(0,y,z)$.
& any point lies on the z-x plane in the form of $P(x,0,z)$.

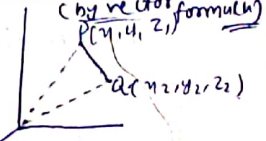
Any point lie on the X-axis in the form of, $P(x,0,0)$.
Similarly, any point lie on the y-axis in the form of $P(0,y,0)$
& any point lie on the z-axis in the form of $P(0,0,z)$.

☆ DISTANCE FORMULA:-

Let $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ are 2 points in space.
then, distance between P & Q is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

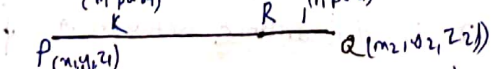
$$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

(by vector formula)



☆ SECTION FORMULA or Division Formula:-(Internally):-

→ Let $P(x_1,y_1,z_1)$ & $P(x_2,y_2,z_2)$ are two points on Space.
If 'R' divides PQ in m:n ratio,
then, co-ordinate of 'R' can also be written as



where R is $\left(\frac{Kx_2+x_1}{K+1}, \frac{Ky_2+y_1}{K+1}, \frac{Kz_2+z_1}{K+1}\right)$

where PR = m part
& PQ = n part.

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☆ Direction Cosine:- (dcs)

→ Let P be an any point on the space.

join 'O' and P.

Suppose, OP makes α, β, γ angles. with OX, OY, OZ respectively.

Then the direction Cosine of OP $\langle \cos\alpha, \cos\beta, \cos\gamma \rangle$

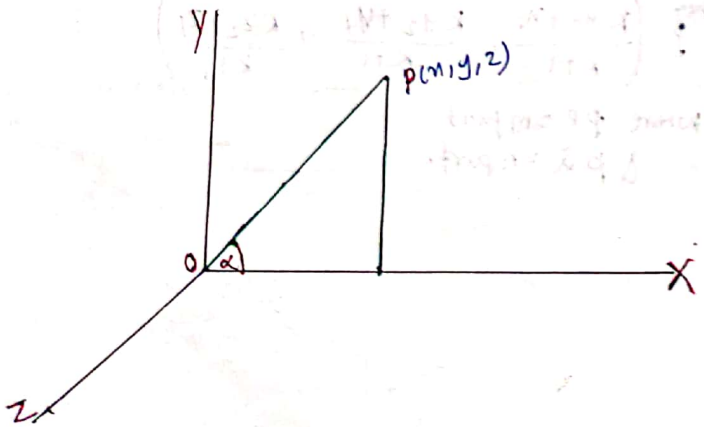
Let $l = \cos\alpha$,
$m = \cos\beta$
$n = \cos\gamma$

the direction Cosine of OP are $\langle l, m, n \rangle$.

☆ THEOREM:-

→ Let P(x, y, z) be any point and $\langle l, m, n \rangle$ are d.c.s of OP then, $x = lr$
$y = mr$
$z = nr$

Where r is the distance between 'O' & 'P'.



let OP makes α, β, γ angles OX, OY, OZ.

from the figure $\cos\alpha = \frac{OQ}{OP}$.

$\cos\alpha = \frac{x}{r}$

$\Rightarrow l = \frac{x}{r}$

$\Rightarrow x = lr$.

similarly $\Rightarrow y = mr$

$z = nr$.

$x^2 + y^2 + z^2 = l^2r^2 + m^2r^2 + n^2r^2$

$\Rightarrow x^2 + y^2 + z^2 = r^2(l^2 + m^2 + n^2)$.

$\Rightarrow l^2 + m^2 + n^2 = \frac{x^2 + y^2 + z^2}{r^2}$

$\Rightarrow l^2 + m^2 + n^2 = \frac{r^2}{r^2}$ =1

$\Rightarrow \boxed{l^2 + m^2 + n^2 = 1}$.

☆ Direction Cosine of X-axis:-

→ Since X-axis makes angle 0°, 90°, 90° with OX, OY, OZ respectively. i.e $\alpha = 0$
$\beta = 90°$
$\gamma = 90°$.

$\Rightarrow \cos\alpha = \cos 0° = 1$
$\Rightarrow \cos\beta = \cos 90° = 0$
$\Rightarrow \cos\gamma = \cos 90° = 0$

So Direction Cosine of X-axis is $= \langle \cos\alpha, \cos\beta, \cos\gamma \rangle$
$= \langle 1, 0, 0 \rangle$.

similarly direction cosine of Y-axis are $\langle 0, 1, 0 \rangle$
& direction Cosine of Z-axis are $\langle 0, 0, 1 \rangle$.

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☆ Direction Ratio :- (d.r.s) :-

→ Any set of 3 number which are proportional to direction cosine are called direction ratios.

let $\langle l, m, n \rangle$ be the direction cosine and the 3 numbers $[a, b, c]$ are proportional to d.c.s $\langle l, m, n \rangle$.

$$\Rightarrow \boxed{\frac{l}{a} = \frac{m}{b} = \frac{n}{c}}$$

$\therefore \langle a, b, c \rangle$ is called direction ratio.

☆ Relation between d.c.s and d.r.s :-

→ from the above relation we get

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = K$$

$$\Rightarrow l = aK$$
$$m = bK$$
$$n = cK$$

Again we can know that

$$l^2 + m^2 + n^2 = 1.$$
$$\Rightarrow a^2K^2 + b^2K^2 + c^2K^2 = 1$$
$$\Rightarrow K^2(a^2 + b^2 + c^2) = 1$$
$$\Rightarrow K^2 = \frac{1}{a^2 + b^2 + c^2}$$
$$\Rightarrow K = \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$\Rightarrow l = aK$.

$$\Rightarrow l = \pm a \cdot \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

~~Similarly~~

Similarly $m = bK$

$$\Rightarrow m = \pm b \cdot \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

& $n = cK$

$$\Rightarrow n = \pm c \cdot \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

taking +ve or -ve sign

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$
$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$
$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Imp

☆ Note :-

(i) Direction ratios of a line are infinite. (Because are infinite).

(ii) Direction cosine of a line are unique. (∵ it's create one angle)

(iii) Direction cosine (d.c.s) of a line is also called as direction ratios (d.r.s).

(iv) i.e d.c.s always a d.r.s but the convert ~~is not~~ it need not be true.

d.r.s $[a, b, c]$ becomes a d.c.s if and only $a^2 + b^2 + c^2 = 1$

☆ THEOREM :-

(v) The d.r.s of a line passing through 2 points (x_1, y_1, z_1) and (x_2, y_2, z_2) are $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$.

☆ Projection of line segment on another line :-

→ Let AB be a line segment and 'l' is a line, then projection of AB on L is $AB\cos\theta$, where θ is angle between $\overline{AB}$ & L.

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☆ Angle between two lines (when d.c.s are given):-

→ Let L_1 & L_2 are two lines, whose d.c.s are $\langle l_1, m_1, n_1 \rangle$ and $\langle l_2, m_2, n_2 \rangle$ respectively, and θ be angle between them, then

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

☆ Angle between two lines (when d.r.s are given):-

→ Let L_1 & L_2 are 2 lines whose d.r.s are $\langle a_1, b_1, c_1 \rangle$ & $\langle a_2, b_2, c_2 \rangle$ respectively, θ then

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Proof: From the above relation

$$l_1 = \frac{a_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}$$

& $$l_2 = \frac{a_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

From the above relation

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$= \frac{a_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} \cdot \frac{a_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}} + \frac{b_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} \cdot \frac{b_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}} + \frac{c_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} \cdot \frac{c_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\Rightarrow \cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\Rightarrow \cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2 .$$

☆ Case-1:-

If two lines are perpendicular

i.e $\theta = 90°$

$$\Rightarrow \cos 90° = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\Rightarrow l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$$

☆ Case-2:-

→ If two lines are parallel,

then, $l_1 = l_2$, $m_1 = m_2$, $n_1 = n_2$.

Again, from the relation, by d.r.s.

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

☆ Case-1:-

If two lines are perpendicular

$\theta = 90°$

$$\Rightarrow a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

☆ Case-2:-

If two lines are parallel,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

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Q1 Find the direction cosines of the line which is perpendicular to the lines whose direction ratios are $\langle 1,-2,3\rangle$ and $\langle 2,2,1\rangle$.

Sol Let the line 'L' whose d.r.s $\langle a,b,c\rangle$, which is $\perp^r$ to the lines whose d.r.s are $L_1\langle 1,-2,3\rangle$ & $L_2\langle 2,2,1\rangle$.

By $\perp^r$ condition $a\cdot 1+b(-2)+c\cdot 3=0$

& $a\cdot 2+b\cdot 2+c\cdot 1=0$

$$\frac{a}{b_1c_2-b_2c_1}=\frac{b}{c_1a_2-c_2a_1}=\frac{c}{a_1b_2-a_2b_1}$$

$$\Rightarrow \frac{a}{(-2)1-2\cdot 3}=\frac{b}{3\cdot 2-1\cdot 1}=\frac{c}{1\cdot 2-(2\cdot(-2))}$$

$$\therefore \frac{a}{-8}=\frac{b}{5}=\frac{c}{6}=k \quad (say) \quad \text{--(1)}$$

$\Rightarrow a=-8k$

$b=5k$

$c=6k$

$$l=\frac{a}{\sqrt{a^2+b^2+c^2}}=\frac{-8k}{\sqrt{(-8k)^2+(5k)^2+(6k)^2}}=\frac{-8k}{\sqrt{k^2(64+25+36)}}=\frac{-8}{5\sqrt{5}}$$

$$\Rightarrow m=\frac{5k}{\sqrt{a^2+b^2+c^2}}=\frac{5k}{k\cdot 5\sqrt{5}}=\frac{1}{\sqrt{5}},$$

$$n=\frac{6k}{k\cdot 5\sqrt{5}}=\frac{6}{5\sqrt{5}}.$$

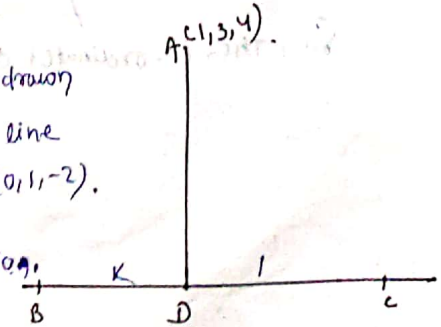
Alternative:

Q2 Find the co-ordinates of the foot of the perpendicular drawn from the point A(1,3,4) to the line joining the points B(3,0,-1) and C(0,1,-2).

Sol Let D be the foot of the $\perp^r$ drawn from the point A(1,3,4) to the line joining the points B(3,0,-1) & (0,1,-2).

and D divides $\overline{BC}$ in K:1 ratio.

where $K=\frac{m}{1=n}$.



Co-ordinate of 'D' are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

$$\left(\frac{K\cdot 0+1\cdot 3}{K+1}, \frac{K\cdot 1+1\cdot 0}{K+1}, \frac{K(-2)+1\cdot(-1)}{K+1}\right)=\left(\frac{3}{K+1}, \frac{K}{K+1}, \frac{-2K-1}{K+1}\right)$$

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d.r.s of $\overline{AD} = \left\langle \frac{3}{k+1} - 1, \frac{k}{k+1} - 3, \frac{-2k-1}{k+1} - 4 \right\rangle$

$\Rightarrow$ d.r.s of $\overline{AD} = \left\langle \frac{2-k}{k+1}, \frac{-2k-3}{k+1}, \frac{-6k-5}{k+1} \right\rangle$

d.r.s of $\overline{BC} = \langle 0-3, 1-0, -2-(-1) \rangle$

$= \langle -3, 1, -1 \rangle$.

Since $AD \perp^r BC$.

By perpendicular d condition.

$$\frac{(2-k)}{k+1}(-3) + \frac{-2k-3}{k+1}(1) + \frac{-6k-5}{k+1}(-1) = 0$$

$$\Rightarrow (2-k)(-3) + (-2k-3)(1) + (-6k-5)(-1) = 0$$

$$\Rightarrow -6+3k - 2k - 3 + 6k + 5 = 0$$

$$\Rightarrow 7k - 4$$

$$\Rightarrow 7k = 4$$

$$\Rightarrow k = \frac{4}{7}$$

So, the co-ordinates of $D = \left(\frac{3}{\frac{4}{7}+1}, \frac{4/7}{\frac{4}{7}+1}, \frac{(-2\times\frac{4}{7}-1)}{\frac{4}{7}+1} \right)$

$$= \left(\frac{21}{11}, \frac{4}{11}, \frac{-15}{11} \right)$$

Now (1) Prove that the points $P(3,2,-4)$, $Q(5,4,-6)$ and $R(9,8,-10)$ are co-linear.

Pf

d.r.s of $\overline{PQ} = \langle (5-3), (4-2), (-6-(-4)) \rangle$

$= \langle 2, 2, -2 \rangle$.

d.r.s of $\overline{QR} = \langle (9-5), (8-4), (-10-(-6)) \rangle$

$= \langle 4, 4, -4 \rangle$ $(\langle a_2, b_2, c_2 \rangle)$

Now $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-2}{-4} = \frac{1}{2}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

$\Rightarrow \overline{PQ} \parallel \overline{QR}$ (parallel).

$\Rightarrow P, Q, R$ are co-linear.

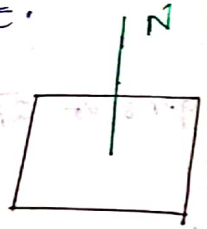
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PLANE

→ **Plane** is a surface such that the line joining any 2 points on the surface lies wholly on it.

→ The general equation of the plane $ax+by+cz+d=0$

→ where a, b, c are **d.r.s** of the **normal** to the plane and d is the constant.



→ $\langle a, b, c \rangle$ are the d.r.s of 'N'.

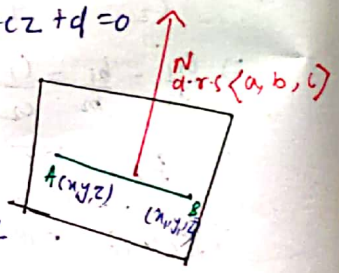
Here 'N' is the normal to the plane.

★ **NORMAL:-**

→ Normal is a line which is perpendicular to the every line lies on the plane.

★ **Equation of plane passing through the point (x_1, y_1, z_1):-**

Suppose the equation of plane is $ax+by+cz+d=0$ and passing through (x_1, y_1, z_1).



$A(x, y, z)$ be any point on the plane, and $B(x_1, y_1, z_1)$ be a given point on the plane.

Now d.r.s of BA are $\langle x-x_1, y-y_1, z-z_1 \rangle$.

Since $N \perp^r BA$.

By $\perp^r$ Condition

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0.$$

→ This is the equation of plane in one point form which is passing through the point (x_1, y_1, z_1) (when d.r.s of the normal are given).

→

→ When **d.c.s** of the normal to the plane are given then, the equation of the plane, we know that d.c.s are always d.r.s.

equation of the plane is $l(x-x_1)+m(y-y_1)+n(z-z_1)=0$.

This is the eqn of the plane in one point form and d.c.s of the normal are given.

e.g. $2x+3y+5z+6=0$.

Here $\langle 2, 3, 5 \rangle$ are the d.r.s of the normal to the given plane.

★ **Note:-**

→ Equation of plane passing through origin $O(0,0,0)$ is $ax+by+cz=0$

Let the general eqn of plane is $ax+by+cz+d=0$,

Once the plane passing through $(0,0,0)$, so the point will be satisfy the equation of the plane,

i.e $a\cdot 0+b\cdot 0+c\cdot 0+d=0$

$\Rightarrow d=0$,

So, general equation becomes $ax+by+cz=0$.

e.g. $\Rightarrow 2x-3y-5z=0$

there is no constant term.

So the plane passing through origin.

★ **Note:-**

→ Every linear equation in x, y, z represents a plane

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Equation of plane passing through three given point :-

Let the eqⁿ of plane is $ax+by+cz+d=0$.

and passing through (x_1,y_1,z_1), (x_2,y_2,z_2) and (x_3,y_3,z_3) : is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0.$$

Note :-

→ In equation of plane there are 3 arbitary constant.

$ax+by+cz+d=0$

$\Rightarrow ax+by+cz=-d$

$\Rightarrow \left(\frac{a}{-d}\right)x+\left(\frac{b}{-d}\right)y+\left(\frac{c}{-d}\right)z=1.$

$\Rightarrow Ax+By+Cz=1$

3 arbitary constant

where $\frac{a}{-d}=A$, $\frac{b}{-d}=B$, $\frac{c}{-d}=C$

Equation of plane in intricept form :-

→ A plane makes X-intercept is 'a', Y-intercept is 'b' & Z-intercept is 'c'.

∴ The plane passes through (x_1,y_1,z_1) $(a,0,0)$, (x_2,y_2,z_2) $(0,b,0)$ & (x_3,y_3,z_3) $(0,0,c)$.

So equation of the plane is

$$\begin{vmatrix} x-a & y-0 & z-0 \\ 0-a & b-0 & 0-0 \\ 0-a & 0-0 & c-0 \end{vmatrix} = 0 \Rightarrow$$

$$\Rightarrow \begin{vmatrix} x-a & y & z \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix}$$

$\Rightarrow (x-a)(bc-0) - y(-ac-0) + z(0+ab) = 0.$

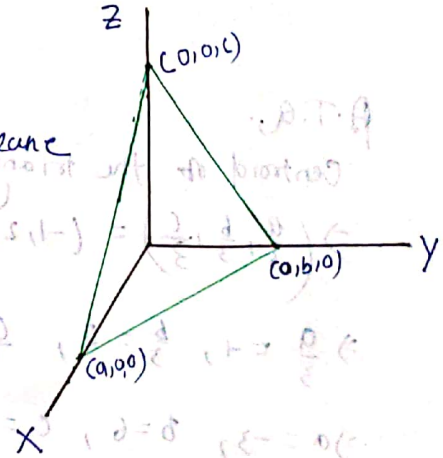
$\Rightarrow bcx - abc + yac + z.ab = 0.$

$\Rightarrow bcx+acy+abz = abc$

$\Rightarrow \frac{bcx}{abc} + \frac{acy}{abc} + \frac{abz}{abc} = \frac{abc}{abc}$ (dividing abc on both the side).

$\Rightarrow \boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.}$

Which is the equation of plane in intercept form.



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Imp.

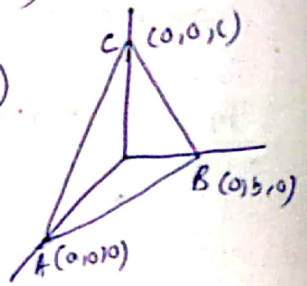
Q. A plane meets the co-ordinate axes at A, B, C respectively. If the centroid of the ΔABC is $(-1, 2, 5)$. Find the eqn of the plane.

Sol: Let, plane meets the x-axis, y-axis & z axis are A, B, C respectively.

Suppose, Co-ordinate of 'A' are $(a, 0, 0)$,
Co-ordinate of 'B' are $(0, b, 0)$.
& Coordinate of 'C' are $(0, 0, c)$.

So centroid of the ΔABC are $\left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$

$= \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$



A.T.Q.,
Centroid of the triangle is $(-1, 2, 5)$

$\Rightarrow \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (-1, 2, 5)$

$\Rightarrow \frac{a}{3} = -1, \frac{b}{3} = 2, \frac{c}{3} = 5$

$\Rightarrow a = -3, b = 6, c = 15$

So, equation of plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\Rightarrow \frac{x}{-3} + \frac{y}{6} + \frac{z}{15} = 1$

$\Rightarrow \frac{-5x + 5y + 2z}{30} = 1$

$\Rightarrow -10x + 5y + 2z = 30$

$\Rightarrow 10x - 5y - 2z + 30 = 0$. (Ans)

☆ Distance of a point from a plane:

→ If $P(x_1, y_1, z_1)$ be a point & the plane $ax + by + cz + d = 0$ then the distance of P from the plane is $D = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$

e.g: Find the distance $(2, 3, 5)$ from the plane $x - 2y + 3z + 5 = 0$.

Given that the point $(2, 3, 5)$ & eqn of plane $= x - 2y + 3z + 5 = 0$
then $a = 1, b = -2, c = 3, d = 5$.

$x_1 = 2$,
$y_1 = 3$,
$z_1 = 5$.

then the distance $D = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$

$= \frac{1 \cdot 2 + (-2 \cdot 3) + (3 \cdot 5) + 5}{\sqrt{(1)^2 + (-2)^2 + (3)^2}}$

$= \frac{2 - 6 + 15 + 5}{\sqrt{14}}$

$= \frac{16}{\sqrt{7} \times \sqrt{2}}$

$= \frac{\sqrt{2} \times 8}{\sqrt{7} \times \sqrt{2}}$

$= \frac{8\sqrt{2}}{\sqrt{14}}$ (Ans).

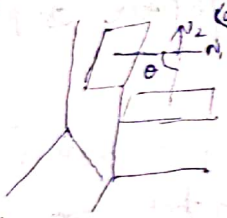
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☆ Angle between 2 planes:-

→ Angle between 2 planes means angle between their normal.

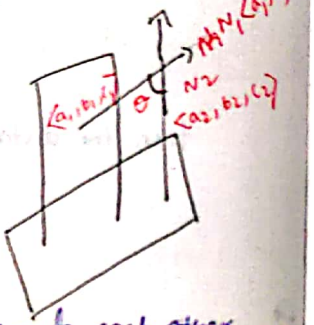
Let $a_1x+b_1y+c_1z+d_1=0$

& $a_2x+b_2y+c_2z+d_2=0$.



Let θ be the angle between 2 normals N_1 & N_2.

Then $\cos\theta = \dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$



☆ Case-1:-

→ If $\theta=90°$, i.e two planes are perpendicular to each other

then $a_1a_2+b_1b_2+c_1c_2=0$.

☆ Case-2:-

→ If $\theta=0°$ two planes are parallel i.e $\theta=0°$.

⇒ then, $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$.

☆ Case-3:-

If $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}=\dfrac{d_1}{d_2}$.

then 2 planes are ~~Co-linear~~ identical.

☆ Equation of plane in normal form:-

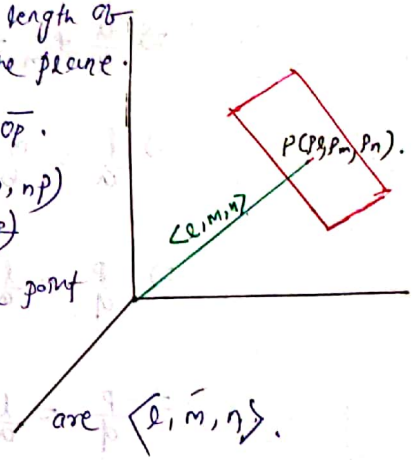
From the figure $P=\overline{OP}$ is the length of perpendicular from the origin to the plane.

and $\langle l,m,n\rangle$ are the d.c.s of $\overline{OP}$.

So co-ordinate of P are (lp, mp, np)

Now, the plane passes through the point $P(lp,mp,np)$

and d.c.s of the normal are $\langle l,m,n\rangle$.



So equation of plane is

$$l(x-lp)+m(y-mp)+n(z-np)=0$$

$$\Rightarrow lx-l^2p+my-m^2p+nz-n^2p=0$$

$$\Rightarrow lx+my+nz-(l^2+m^2+n^2)p=0$$

$$\Rightarrow lx+my+nz=(l^2+m^2+n^2)p$$

$$\Rightarrow \boxed{lx+my+nz=p} \quad (\because l^2+m^2+n^2=1).$$

→ This is the eqn of plane in normal form.

☆ Transformation of general form to normal form:-

Let $ax+by+cz+d=0$ is generating form of eqn of plane and its corresponding normal form is

$lx+my+nz=p$.

~~So these~~

So these 2 equation of plane are same.

$$\Rightarrow \frac{a}{l} = \frac{b}{m} = \frac{c}{n} = \frac{d}{-p}$$

$$\Rightarrow \frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{-p}{d}$$

$$\Rightarrow \frac{-p}{d} = \frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \frac{-p}{d} = \frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{\pm 1}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \frac{p}{-d} = \frac{1}{\pm\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow p = \frac{-d}{\pm\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow l = \frac{a}{\pm\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow m = \frac{b}{\pm\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow n = \frac{c}{\pm\sqrt{a^2+b^2+c^2}}.$$

$\Rightarrow$ Since p is +ve, if d is +ve then, we choose -ve sign of $\sqrt{a^2+b^2+c^2}$.

or d is -ve then, we choose the sign of $\sqrt{a^2+b^2+c^2}$

So its normal form is

$$\frac{a}{\pm\sqrt{a^2+b^2+c^2}}x + \frac{b}{\pm\sqrt{a^2+b^2+c^2}}y + \frac{c}{\pm\sqrt{a^2+b^2+c^2}}z = p.$$

eg Convert the general form or equation of plane $2x+3y+5z+6=0$ to normal form:

Short trick

$$2x+3y+5z = -6$$
$$-2x-3y-5z = 6$$

$$\frac{-2\cdot x}{\sqrt{(-2)^2+(-3)^2+(-5)^2}} - \frac{3\cdot y}{\sqrt{(-2)^2+(-3)^2+(-5)^2}} - \frac{5\cdot z}{\sqrt{(-2)^2+(-3)^2+(-5)^2}} = \frac{6}{\sqrt{(-2)^2+(-3)^2+(-5)^2}}$$

$$\Rightarrow \frac{-2x}{\sqrt{38}} - \frac{3y}{\sqrt{38}} - \frac{5z}{\sqrt{38}} = \frac{6}{\sqrt{38}}.$$

★ Transformation of general form to intercept form:

→ Let $ax+by+cz+d=0$. the general form of eqn of plane.

Now $ax+by+cz+d=0$

$$\Rightarrow ax+by+cz = -d$$

$$\Rightarrow \frac{ax}{-d} + \frac{by}{-d} + \frac{cz}{-d} = 1.$$

$$\Rightarrow \frac{x}{(-d/a)} + \frac{by}{(-d/b)} + \frac{z}{(-d/c)} = 1.$$

$$\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1$$

where $A = -d/a$
$B = -d/b$
$C = -d/c$.

$\left(-\frac{d}{a}, 0, 0\right)$, $\left(0, \frac{d}{b}, 0\right)$, $\left(0, 0, -\frac{d}{c}\right)$

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e.g:
→ $2x+3y+5z+7=0$ convert to intercept form.

$2x+3y+5z=-7$

$\Rightarrow \frac{2x}{-7}+\frac{3y}{-7}+\frac{5z}{-7}=1$

$\Rightarrow \frac{x}{(-7/2)}+\frac{y}{(-7/3)}+\frac{z}{(-7/5)}=1$

☆ EQUATION OF CO-ORDINATE PLANE:-

(1) ~~Equation of XY-plane~~:-

(2) equation of any XY plane:

$\Rightarrow z=0$

(1) equation of YZ plane

$\Rightarrow x=0$

(3) equation of ZX plane

$\Rightarrow y=0$.

☆ IMAGE OF A POINT w.r.t a plane:-

→ Image of a point $P(\alpha,\beta,\gamma)$ w.r.t XY plane is $Q(\alpha,\beta,-\gamma)$.

Similarly the image of $P(\alpha,\beta,\gamma)$ w.r.t YZ plane is $Q(-\alpha,\beta,\gamma)$.

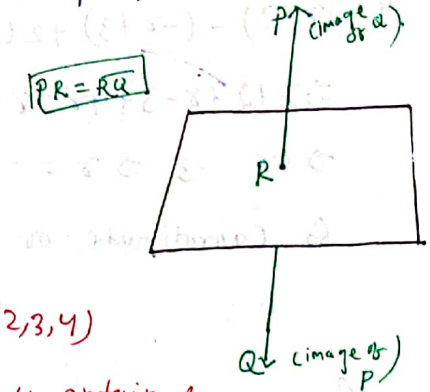
& the image of $P(\alpha,\beta,\gamma)$ w.r.t ZX plane is $Q(\alpha,-\beta,\gamma)$.

☆ NOTE:-

→ The image of the point 'P' w.r.t a given plane is Q means the distance between P & plane is equal to the distance between Q & plane.

i.e in other words,

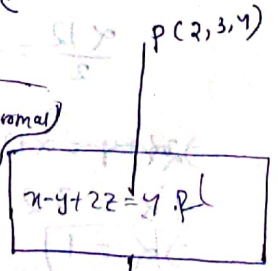
The line segment $\overline{PQ}$ is perpendicular to the given plane.



Q. Find the image of the point (2,3,4) w.r.t the plane $x-y+2z=4$ obtain the foot of the perpendicular from P to plane and the corresponding perpendicular distance.

Sol: Let P be (2,3,4) be a point & Q be the image of P w.r.t $x-y+2z=4$.

Since $\overline{PQ} \perp^r$ the given plane (i.e PQ is also a normal to the plane) so, d.r.s of PQ $\langle 1,-1,2\rangle$



Since $\overline{PQ}$ passes through (2,3,4)

So, eqn of PQ, $\frac{x-2}{1}=\frac{y-3}{-1}=\frac{z-4}{2}$

$\Rightarrow \frac{x-2}{1}=\frac{y-3}{-1}=\frac{z-4}{2}=r$ (say)

$x=r+2$

$y=-r+3$

$z=2r+4$.

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Since R lies on $\overleftrightarrow{PQ}$

So, co-ordinates of R are $(r+2, -r+3, 2r+4)$

moreover R lies on the given plane therefore R satisfy the eqn of plane.

i.e $(r+2) - (-r+3) + 2(2r+4) = 4$

$\Rightarrow r+2+r-3+4r+8 = 4$

$\Rightarrow 6r = -3 \Rightarrow r = -\frac{3}{6} = -\frac{1}{2}$.

So Coordinate of R (midpoint) are

$\left(-\frac{1}{2}+2, -\frac{1}{2}+3, 2\cdot\frac{-1}{2}+4\right)$

$= \left(\frac{3}{2}, \frac{7}{2}, 3\right)$.

Suppose Co-ordinate of Q are (α, β, γ)

Now using midpoint formula

Co-ordinate of R are $= \left(\frac{\alpha+2}{2}, \frac{\beta+3}{2}, \frac{\gamma+4}{2}\right)$

$\frac{\alpha+2}{2} = \frac{3}{2}$; $\frac{\beta+3}{2} = \frac{7}{2}$; $\frac{\gamma+4}{2} = 3$

$\Rightarrow \alpha+2 = \frac{6}{2}$, $\beta+3 = 7$, $\gamma+4 = 6$

$\Rightarrow \alpha = 3-2$ $\Rightarrow \boxed{\beta = 4}$, $\Rightarrow \boxed{\gamma = 2}$

$\Rightarrow \boxed{\alpha = 1}$

So the Co-ordinate of Q' ~~is the Co-ordinate image of the given point P is~~

is $(1, 4, 2)$ (Ans)

★ SYSTEM OF PLANES (FAMILY OF PLANES) :=

Let $a_1x+b_1y+c_1z+d_1=0$,

$a_2x+b_2y+c_2z+d_2=0$ are 2 planes, then the equation of family of planes passing through the line of intersection of two given planes are

$(a_1x+b_1y+c_1z+d_1)+k(a_2x+b_2y+c_2z+d_2)=0$ (where $k \in R$)

★ Position of 2 given point w.r.t a plane :=

Let $ax+by+cz+d=0$ — (1)

be a given plane and $P(x_1, y_1, z_1)$ $Q(x_2, y_2, z_2)$ be two given points. Let $\overleftrightarrow{PQ}$ meet the plane (1) at a point R.

Then the co-ordinates of the point R are, ($\because$ R divides $\overline{PQ}$ in $\lambda:1$ ratio)

$\left(\frac{\lambda x_1+x_2}{1+\lambda}, \frac{\lambda y_1+y_2}{1+\lambda}, \frac{\lambda z_1+z_2}{1+\lambda}\right)$ for some real number $\lambda \neq 1$

Since the point R lines on the plane (1) we have,

$a\left(\frac{\lambda x_1+x_2}{1+\lambda}\right)+b\left(\frac{\lambda y_1+y_2}{1+\lambda}\right)+c\left(\frac{\lambda z_1+z_2}{1+\lambda}\right)+d=0$,

or $\lambda(ax_1+by_1+cz_1+d)+(ax_2+by_2+cz_2+d)=0$.

or $\lambda = -\frac{ax_2+by_2+cz_2+d}{ax_1+by_1+cz_1+d}$

→ if $ax_1+by_1+cz_1+d$ and $ax_2+by_2+cz_2+d$ are same sign. then 2 points lie on the same sign of the plane.

→ if $ax_1+by_1+cz_1+d$ and $ax_2+by_2+cz_2+d$ are opposite sign then 2 points die on the opposite side of the plane.

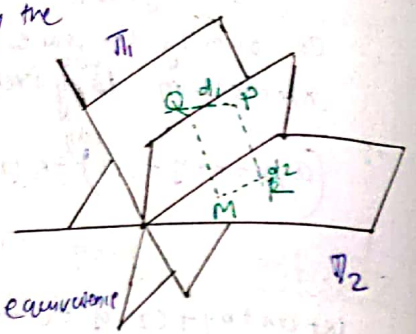
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☆ Equations of planes bisecting the angle between two given planes :=

Consider two planes given by the equation

$a_1x+b_1y+c_1z+d_1=0$

and $a_2x+b_2y+c_2z+d_2=0$



if $P(x,y,z)$ is any point on the bisector plane, then it is equivalent from both the given plane.

→ one of the plane given by the internal bisector and the other is external bisector of the angle between the given planes. TO distinguish between these 2 planes, we find out the angle between arc of the bisector planes and a given plane. If measure of this angle is greater than $45°$, The concerned angle is obtuse and the bisector plane is the external bisector. Otherwise, it is the internal bisector. ~~We will illustrate this in example~~

☆ Equation of plane parallel to a given plane :=

Let $ax+by+cz+d=0$ be a plane — (1)

Then the eqn of plane parallel to eqn (1) is $ax+by+cz+k=0$.

☆ Distance between 2 parallel plane :=

Let $ax+by+cz+d=0$ & $ax+by+cz+k=0$ are two parallel planes

The distance between 2 parallel planes $= \left|\frac{d-k}{\sqrt{a^2+b^2+c^2}}\right|$.

✓ Imp eg:

Find the distance between 2 planes.

when $2x+3y+5z+6=0$

& $2x+3y+5z+9=0$.

Sol^n Distance between 2 planes is $\left|\frac{6-9}{\sqrt{2^2+3^2+5^2}}\right|$ $\left\{\because \frac{d-k}{\sqrt{a^2+b^2+c^2}}\right\}$

$\Rightarrow \left|\frac{-3}{\sqrt{4+9+25}}\right|$

$= \left|\frac{-3}{\sqrt{38}}\right|$ unit.

Q Find the distance between 2 Parallel planes.

where $2x+3y+5z+6=0$

& $8x+12y+20z+16=0$.

Sol^n Now two planes are $2x+3y+5z+6=0$

& $8x+12y+20z+10=0$.

$\Rightarrow 4(2x+3y+5z+\frac{10}{4})=0$

$\Rightarrow 2x+3y+5z+\frac{10}{4}=0$

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Two parallel planes are:

$2x+3y+5z+6=0$ & $2x+3y+5z+\frac{10}{4}=0$ $\left\{\begin{matrix} ax+by+cz+d=0 \\ ax+by+cz+k=0 \end{matrix}\right\}$ —

So, distance between 2 parallel plane is

$$\left|\frac{6-\frac{10}{4}}{\sqrt{2^2+3^2+5^2}}\right| = \left|\frac{\frac{24-10}{4}}{\sqrt{38}}\right| \quad \left\{p=\frac{d-k}{\sqrt{a^2+b^2+c^2}}\right\}$$

$$\Rightarrow \left|\frac{14}{4\sqrt{38}}\right|$$

$$= \frac{7}{2\sqrt{38}} \text{ unit.}$$

★ NOTE:- ⓐ

→ Suppose 2 plane $a_1x+by+c_1z+d_1=0$ & $a_2x+b_2y+c_2z+d_2=0$ are parallel.

(i) If $a_1=a_2$, $b_1=b_2$, & $c_1=c_2$

then distance between 2 parallel plane

$$= \left|\frac{d_1-d_2}{\sqrt{a_1^2+b_1^2+c_1^2}}\right|$$

(ii) If (a_1, b_1, c_1) & (a_2, b_2, c_2) are proportional.

To make the co-efficient of $x, y,$ & z of the 2 equation are equal. Then distance between them,

$$= \frac{\text{difference between their Constant}}{\sqrt{(\text{Co-efficient of } x)^2+(\text{co-efficient of } y)^2+(\text{Co-efficient of } z)^2}}$$

STRAIGHT LINE

★ CARTESIAN EQUATION OF A LINE:-

→ Since the straight line is the intersection of 2 planes, a pair of an equation of 1st degree in x, y & z such as

$a_1x+b_1y+c_1z+d_1=0$

$a_2x+b_2y+c_2z+d_2=0$.

is sufficient to determine a line. If exist a values of x, y, z satisfies the 2 equations then the point with co-ordinates (x, y, z) lies on these planes and hence give the co-ordinates of a point on the line of intersection of the 2 planes. Conversely; if we take a point on the line of intersection of these two planes, then it will satisfy the eqn of both planes. Hence the combined eqn of a pair of planes (not parallel) can always be considered as the general eqn of a straight line. The form of eqn of straight line is otherwise known as unsymmetric form of eqn of line.

→ The unsymmetric form of a line can be written as either

$a_1x+b_1y+c_1z+d_1=0$
$a_2x+b_2y+c_2z+d_2=0$ } —(i)

or $a_1x+b_1y+c_1z+d_1=0 = a_2x+b_2y+c_2z+d_2$

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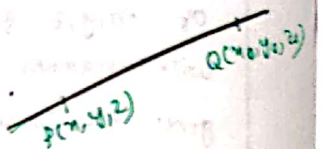
★ Symmetric form of an eqⁿ of line:

Consider a line L passing through a point $Q(x_0, y_0, z_0)$ and having direction ratio $\langle a, b, c \rangle$.

Then for any variable point $P(x, y, z)$ on the line L, the direction ratio of $\overleftrightarrow{PQ}$ are $\langle x - x_0, y - y_0, z - z_0 \rangle$

Hence we get,

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$



which are the eqns of the line in symmetric form.

Now, equating each of the above fraction as a parameter r, we get

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} = r$$

Hence co-ordinates of any variable point on the line L are given by $(x_0 + ar, y_0 + br, z_0 + cr)$ for some real value of the parameter r.

Since, any point on the line, whose co-ordinates are in the form of $(x_0 + ar, y_0 + br, z_0 + cr)$

eg: $\left(\frac{x-1}{2} = \frac{y-3}{5} = \frac{z-4}{6}\right)$

→ This is the eqⁿ of the line in symmetric form where $\langle 2, 5, 6 \rangle$ are d.r.s of the given line & the point $(1, 3, 4)$ is the point which lies on the line e.g. (line passing through the point $(1, 3, 4)$

eg:

★ Two point form:

→ If a line passes through a given point $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$. Then the eqn of line is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

or $\frac{x - x_2}{x_2 - x_1} = \frac{y - y_2}{y_2 - y} = \frac{z - z_2}{z_2 - z_1}$

Since the line passes through (x_1, y_1, z_1) & (x_2, y_2, z_2) So d.r.s of the line are $\langle (x_2 - x_1), (y_2 - y_1), (z_2 - z_1) \rangle$ $(\because \langle a, b, c \rangle)$

again the line passes through (x_1, y_1, z_1) $(i.e.\ (x_0, y_0, z_0))$

So eqn of line is $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

Imp:

★ Transformation of unsymmetrical form to symmetrical form:

Consider the unsymmetrical form of eqn of line, represented by the pair of question, equation.

$$a_1x + b_1y + c_1z + d_1 = 0$$
$$a_2x + b_2y + c_2z + d_2 = 0.$$

To reduce it to symmetric form, we require the direction ratios of the line and the co-ordinates of a point on it. Now if $\langle a, b, c \rangle$ are the direction ratios of the line, then the line being common to both the planes, the normals to the both planes are perpendicular to the line. Hence by condition of perpendicularity,

$$aa_1 + bb_1 + cc_1 = 0 \ \&\ aa_2 + bb_2 + cc_2 = 0.$$

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$$\frac{a}{b_1c_2-c_1b_2} = \frac{b}{c_1a_2-a_1c_2} = \frac{c}{a_1b_2-b_1a_2}$$

Let p be a point on the line whose z-co-ordinate is zero.
i.e - the co-ordinate of p are $(\alpha, \beta, 0)$

This point lies on both the planes, so this point will be satisfy the equation of both the planes.

$\Rightarrow a_1\alpha + b_1\beta + d_1 = 0$

$a_2\alpha + b_2\beta + d_2 = 0$.

$$\alpha = \frac{b_1d_2 - b_2d_1}{a_1b_2 - a_2b_1}, \quad \beta = \frac{d_1a_2 - d_2a_1}{a_1b_2 - a_2b_1}$$

So equation of the line in symmetric form is

$$\frac{x-\alpha}{b_1c_2 - b_2c_1} = \frac{y-\beta}{c_1a_2 - a_1c_2} = \frac{z-0}{a_1b_2 - a_2b_1}$$

Condition for a line to lie on a plane :-

Consider the line

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

and the plane

$$Ax + By + Cz + D = 0$$

If the line lies on the plane then the normal to the plane is normal to the line and hence by condition of perpendicularity

$$Aa + Bb + Cc = 0$$

But this condition also implies that the line may be partial parallel to the plane (as in that case also the normal to the plane is perpendicular to the line).

So, it along with this we take the condition that at least one point of the line lies on the plane, then the line will lie on the plane. The point (x_0, y_0, z_0) being given point on the line, lies on the plane, if

$$Ax_0 + By_0 + Cz_0 + D = 0 .$$

Hence the required conditions are.

(i) $Aa + Bb + Cc = 0$.

(ii) $Ax_0 + By_0 + Cz_0 + D = 0$.

Condition for two lines to be coplanar :-

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

and $$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

The required condition is

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

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✱ Angle between line & plane:

Consider a line $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

and a plane $Ax+By+Cz+D=0$

Angle between line and plane is given by sinθ.

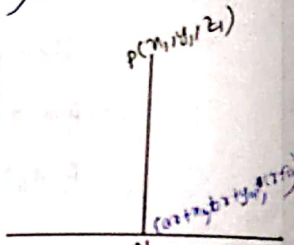
$$\sin\theta = \left(\frac{Aa+Bb+Cc}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{a^2+b^2+c^2}}\right)$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{Aa+Bb+Cc}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{a^2+b^2+c^2}}\right).$$

✱ Distance of a Point from a line:

Let the line be

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}.$$



and the point P be (x_1, y_1, z_1). If 'N' be the foot of the perpendicular drawn from P on L, then N being a point on L, its co-ordinates are (x_0+ar, y_0+br, z_0+cr), for some value of the parameter r, Now the direction ratios of $\overline{PN}$ are $\langle x_0+ar-x_1, y_0+br-y_1, z_0+cr-z_1\rangle$ and those of L are $\langle a, b, c\rangle$. So by condition of perpendicularity,

$$a(x_0+ar-x_1)+b(y_0+br-y_1)+c(z_0+cr-z_1)=0,$$

which gives the value of r and hence the co-ordinates of N.

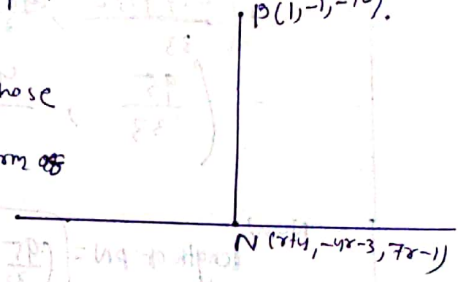
$$|PN| = \sqrt{(ar+x_0-x_1)^2+(br+y_0-y_1)^2+(cr+z_0-z_1)^2}$$

Q) Find the distance of the point $(1,-1,-10)$ from the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$

Sol) Let 'L' be a line whose equation is $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$.

and P be a point whose co-ordinates are $(1,-1,-10)$.

& N being the foot of the perpendicular.



Any point on the line L whose co-ordinates are in the form $(r+4, -4r-3, 7r-1)$.

Since N lie on the L,

Co, co-ordinate of N are $(r+4, -4r-3, 7r-1)$.

d.r.s. of PN are

$$\langle r+4-1, -4r-3+1, 7r-1+10\rangle$$

$$= \langle r+3, -4r-2, 7r+9\rangle$$

Since, PN is perpendicular to L.

By perpendicular condition, $1\cdot(r+3)+(-4)\cdot(-4r-2)+7\cdot(7r+9)=0$.

$$\Rightarrow r+3+16r+8+49r+63=0$$

$$\Rightarrow 66r+74=0$$

$$\Rightarrow 66r=-74$$

$$\Rightarrow r = -\frac{74}{66} = -\frac{37}{33}$$

$$\Rightarrow \boxed{r = -\frac{37}{33}}.$$

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So coordinate of N are.

$(r+4, -4r-3, 7r-1)$

$\Rightarrow \left(\frac{-37}{33}+4, -4\cdot\frac{-37}{33}-3, 7\cdot\frac{-37}{33}-1\right)$

$\Rightarrow \left(\frac{-37+132}{33}, \frac{148-99}{33}, \frac{-259-33}{33}\right)$

$= \left(\frac{95}{33}, \frac{49}{33}, \frac{-292}{33}\right)$

Now length of $PN = \sqrt{\left(\frac{95}{33}-1\right)^2+\left(\frac{49}{33}+1\right)^2+\left(\frac{-292}{33}+10\right)^2}$

$= \sqrt{\left(\frac{95-33}{33}\right)^2+\left(\frac{49+33}{33}\right)^2+\left(\frac{-292+330}{33}\right)^2}$

$= \sqrt{\left(\frac{62}{33}\right)^2+\left(\frac{82}{33}\right)^2+\left(\frac{38}{33}\right)^2}$

$= \frac{\sqrt{(62)^2+(82)^2+(38)^2}}{33}$

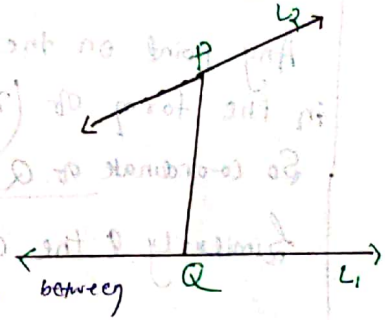
$= \frac{\sqrt{3844+6724+1444}}{33}$

$= \frac{\sqrt{11912}}{33} \quad \frac{\sqrt{12012}}{33}$

$=$

☆ Skew lines:-

→ Two lines are neither parallel nor intercept are called skew lines.
Such lines possess a common perpendicular, which is the shortest distance between them.



Here L_1 & L_2 are neither parallel nor intercept. So L_1 & L_2 are skew lines.

PQ is perpendicular to both L_1 & L_2.

Therefore $\overline{PQ}$ is the shortest distance between L_1 & L_2.

~~Let $\frac{x-x_1}{a_1}$~~

Eqn of $L_1 = \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

& eqn of $L_2 = \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

let $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} = r_1$ (say). ——— (I)

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2} = r_2$ (say) ——— (II)

Suppose d.r.s of PQ are $\langle a, b, c \rangle$.

Since, $\overline{PQ}$ is perpendicular to L_1 by perpendicular condition $aa_1+bb_1+cc_1=0$

and $aa_2+bb_2+cc_2=0$

$\Rightarrow \frac{a}{b_1c_2-b_2c_1} = \frac{b}{c_1a_2-c_2a_1} = \frac{c}{a_1b_2-a_2b_1}$

So d.r.s of PQ are $\langle b_1c_2-b_2c_1, c_1a_2-c_2a_1, a_1b_2-a_2b_1 \rangle$.

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From the equation (1)

$x = x_1 + a_1 r_1$

$y = y_1 + b_1 r_1$

$z = z_1 + c_1 r_1$

Any point on the line L_1 whose co-ordinates are in the form of $(x_1 + a_1 r_1, y_1 + b_1 r_1, z_1 + c_1 r_1)$

So co-ordinate of Q are $(x_1 + a_1 r_1, y_1 + b_1 r_1, z_1 + c_1 r_1)$

Similarly the co-ordinate of P are

$(x_2 + a_2 r_2, y_2 + b_2 r_2, z_2 + c_2 r_2)$

dir. r of $PQ = \langle x_1 - x_2 + a_1 r_1 - a_2 r_2, y_1 - y_2 + b_1 r_1 - b_2 r_2, z_1 - z_2 + c_1 r_1 - c_2 r_2 \rangle$

Since $PQ \perp^r$ to L_1 by perpendicular condition,

$$a_1(x_1 - x_2 + a_1 r_1 - a_2 r_2) + b_1(y_1 - y_2 + b_1 r_1 - b_2 r_2) + c_1(z_1 - z_2 + c_1 r_1 - c_2 r_2) = 0$$

& $a_2(x_1 - x_2 + a_1 r_1 - a_2 r_2) + b_2(y_1 - y_2 + b_1 r_1 - b_2 r_2) + c_2(z_1 - z_2 + c_1 r_1 - c_2 r_2) = 0$.

Solving these 2 eqn we get the value of r_1 & r_2.

Shortest distance (S.d) =

$$\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2\}}}$$

Q find the shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

& $$\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

Sol^n Here $(x_1, y_1, z_1) = (3, 8, 3)$

$(x_2, y_2, z_2) = (-3, -7, 6)$

$\langle a_1, b_1, c_1 \rangle = \langle 3, -1, 1 \rangle$

$\langle a_2, b_2, c_2 \rangle = \langle -3, 2, 4 \rangle$

Shortest distance (S.d) =

$$\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2\}}}$$

$$= \frac{\begin{vmatrix} -6 & -15 & 3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}}{\sqrt{(-4-2)^2 + (-3-12)^2 + (6-3)^2}}$$

$$= \frac{-6(-4-2) + 15(12+3) + 3(6-3)}{\sqrt{36 + 225 + 9}}$$

$$= \frac{36 + 225 + 9}{\sqrt{36 + 225 + 9}}$$

$$= \frac{270}{\sqrt{270}}$$

$$= \sqrt{270} \text{ unit}$$

$$= 3\sqrt{30} \text{ unit}$$

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Alternative

$Q(x_1, y_1, z_1) = (3r_1 + 3,\ -r_1 + 8,\ r_1 + 3)$

$P(x_2, y_2, z_2) = (-3r_2 - 3,\ -2r_2 - 7,\ 4r_2 + 6)$.

d.r.s of $PQ = \langle 3r_1 + 3r_2 + 6,\ -r_1 - 2r_2 + 15,\ r_1 - 4r_2 - 3 \rangle$

d.r.s of $L_1 = \langle 3, -1, 1 \rangle$

d.r.s of $L_2 = \langle -3, 2, 4 \rangle$.

Since $\overline{PQ} \perp^r L_1$, $\overline{PQ} \perp^r L_2$

By $\perp^r$ condition

$3(3r_1 + 3r_2 + 6) + (-1)\cdot(-r_1 - 2r_2 + 15) + 1\cdot(r_1 - 4r_2 - 3) = 0$

$\Rightarrow -3(3r_1 + 3r_2 + 6) + 2(-r_1 - 2r_2 + 15) + 4(r_1 - 4r_2 - 3) = 0$

$11r_1 + 7r_2 = 0$ —①

$-7r_1 - 29r_2 = 0$ —②

eqn ① × 11 = $77r_1 + 49r_2 = 0$

eqn ② × 7 = $-77r_1 - 319r_2 = 0$.

$-270r_2 = 0$

$\therefore r_2 = 0$.

Putting the value of $r_2 = 0$ in eqn ①.

$11r_1 + 7\cdot 0 = 0$

$\Rightarrow 11r_1 = 0$

$\Rightarrow r_1 = 0$

So Co-ordinate of $Q = (3, 8, 3)$

& Co-ordinate of $P = (-3, -7, 6)$

$|\overline{PQ}| = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2} = \sqrt{36 + 225 + 9} = \sqrt{270}$

$\Rightarrow |PQ| = 3\sqrt{30}$ units.

Q. Find the eqn of plane passing through $(2, 3, -1)$ and parallel to the plane $3x - 4y + 7z = 0$.

Sol^n The required plane is parallel to $3x - 4y + 7z = 0$.

So eqn of the required plane is

$3x - 4y + 7z + k = 0$

plane (2): $ax + by + cz + k = 0$, $3x - 4y + 7z + k = 0$

again the required plane passing through the point $(2, 3, -1)$ so, this point satisfy the eqn of plane.

plane (1): $ax + by + cz + q = 0$, $3x - 4y + 7z = 0$

i.e $3x - 4y + 7z + k = 0$

$\Rightarrow 3\cdot 2 + -4\cdot 3 + 7\cdot(-1) = 0 + k = 0$

$\Rightarrow 6 - 12 - 7 + k = 0$

$\Rightarrow 6 - 19 + k = 0$

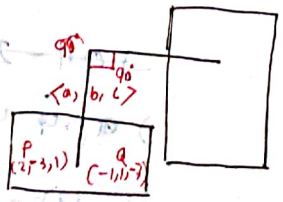
$\Rightarrow 6 - 13 + k = 0$

$\Rightarrow k = 13$.

So the required plane is $3x - 4y + 7z + 13 = 0$.

Q. Find the eqn of plane passing through the point $(2, -3, 1)$ & $(-1, 1, 7)$ and $\perp^r$ to the plane $x - 2y + 5z + 11 = 0$.

Sol^n Let $\langle a, b, c \rangle$ are the d.r.s of the normal to the required plane,



So equation of the plane

$a\cdot(x - 2) + b(y + 3) + c(z - 7)$.

Again this also passing through $(-1, 1, -7)$.

So this point will be satisfy the eqn of plane

i.e $a(-1 - 2) + b(1 + 3) + c(-7 - 1) = 0$

$\Rightarrow -3a + 4b - 8c = 0$ —(*)

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$-3x+4y-8z$.

d.r.s of normal to the given plane are $\langle 1,-2,5\rangle$.

d.r.s of normal to the required plane are $\langle a,b,c\rangle$.

So d by I° condition $a\cdot 1+b(-2)+c\cdot 5=0$.

$\Rightarrow a-2b+5c=0$ ——— (**)

From eqn (i) & (**). we get

$$\frac{a}{b_1c_2-b_2c_1}=\frac{b}{c_1a_2-a_2c_2}=\frac{c}{a_1b_2-a_2b_1}$$

$$\Rightarrow \frac{a}{4\cdot 5-(-2)(-4)}=\frac{b}{(-3)\cdot 1-(-3)\cdot 5}=\frac{c}{-3\cdot(-2)-1\cdot 4}$$

$$\Rightarrow \frac{a}{4}=\frac{b}{7}=\frac{c}{2}.$$

So d.r.s of the normal to the required plane is $\langle 4,7,2\rangle$.

So the eqn of plane is

$$a(x-2)+b(y+3)+c(z-1)=0$$

~~$\Rightarrow 4x-2a+7y+3b$~~

$\Rightarrow 4(x-2)+7(y+3)+2(z-1)=0$

$\Rightarrow 4x-8+7y+21+2z-2=0$.

$\Rightarrow 4x+7y+2z+11=0$

Q. find the eqn of plane passing through the foot of the perpendiculars drawn from $P(a,b,c)$ on the co-ordinate planes.

Sol" Let R, S, T are the foot of the perpendiculars from $P(a,b,c)$ on xy, yz, zx plane respectively.

So co-ordinate of R are $(a,b,0)$

Similarly co-ordinate of S are $(0,b,c)$.

& co-ordinate of T are $(a,0,c)$.

Then eqn of the plane passing through

$R(a,b,0)$ i.e. (x_1,y_1,z_1), $S(0,b,c)$ i.e. (x_2,y_2,z_2) & $T(a,0,c)$ i.e. (x_3,y_3,z_3)

is
$$\begin{vmatrix} x-a & y-b & z-0 \\ 0-a & b-b & c-0 \\ a-a & 0-b & c-0 \end{vmatrix}$$

$$=\begin{vmatrix} x-a & y-b & z \\ -a & 0 & c \\ 0 & -b & c \end{vmatrix}$$

$=x-a(bc)-y(-ac-0)+z(ab-0)$

$=(x-a)(bc)-(y-b)(-ac)+z(ab-0)$

$\Rightarrow xbc-abc+yac+zab-abc$. $\Rightarrow xbc-abc+yac-abc+zab$

$\Rightarrow \frac{xbc}{abc}-\frac{abc}{abc}+\frac{yac}{abc}+\frac{zab}{abc}-\frac{abc}{abc}=0$

$\Rightarrow \frac{x}{a}-1+\frac{y}{b}+\frac{z}{c}-1=0$

$\Rightarrow xbc+yac+zab-2abc=0$

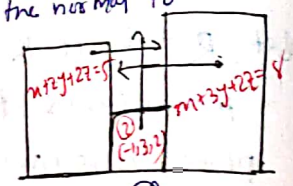
$\Rightarrow \frac{xbc}{abc}+\frac{yac}{abc}+\frac{zab}{abc}-\frac{2abc}{abc}=\frac{0}{abc}$

$\Rightarrow \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=2$ (Ans).

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Q- Find the eqn of plane passing through the point $(-1,3,2)$ $\perp^r$ to the planes $x+2y+2z=5$ & $3x+3y+2z=8$.

Let a,b,c are the d.r.s of the normal to the required plane.



So the eqn of plane is:

$$a(x+1)+b(y-3)+c(z-2)=0 \quad \text{---(1)}$$

But given that the required plane is $\perp^r$ to the

$x+2y+2z=5$

& $3x+3y+2z=8$

So, By by $\perp^r$ condition..

$$a\cdot 1+b\cdot 2+c\cdot 2=0 \quad \text{---(1)}$$

& $a\cdot 3+b\cdot 3+c\cdot 2=0$ ---(2)

from eqn (1) & (2) we get.

$$\frac{a}{b_1c_2-b_2c_1}=\frac{b}{c_1a_2-c_2a_1}=\frac{c}{a_1b_2-a_2b_1}$$

$$\Rightarrow \frac{a}{2\cdot 2-(3\cdot 2)}=\frac{b}{2\cdot 3-3\cdot 2\cdot 1}=\frac{c}{1\cdot 3-3\cdot 2}$$

$$=\frac{a}{-2}=\frac{b}{4}=\frac{c}{-3}$$

So d.r.s of the required plane is $\langle -2,4,-3\rangle$

Now the eqn of required plane is, Now eqn (1) becomes

$$-2(x+1)+4(y-3)-3(z-2)$$

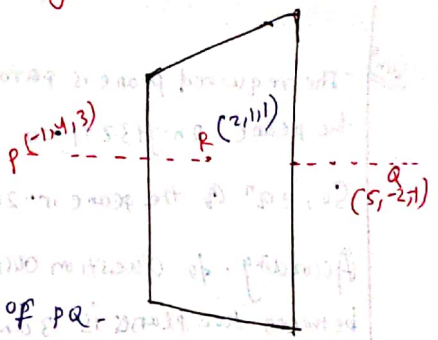
$$\Rightarrow -2x-2+4y-12-3z+6=0 \Rightarrow -2x+4y-3z-8=0$$

$$\Rightarrow 2x-4y+3z+8=0$$

Q Find the eqn of plane bisecting the line segment joining $(-1,4,3)$ & $(5,-2,-1)$ at right angle.

Soln Let the co-ordinates of P are $P(-1,4,3)$

& Coordinates of Q are $Q(5,-2,-1)$



Suppose R is the mid point of PQ.

Since, the plane bisecting the line segment PQ

So R lies on the plane.

So, co-ordinate of R are

$$\left(\frac{-1+5}{2},\frac{4-2}{2},\frac{3-1}{2}\right)$$

$$\Rightarrow R(2,1,1)$$

Since, So PR is perpendicular to the plane,

So PR is normal to the plane

So d.r.s of $\overline{PR}=\langle 2-(-1),1-4,1-3\rangle$

$=\langle 3,-3,-2\rangle$

~~So eqn of the plane~~

Since the plane passing through R are $(2,1,1)$ and and d.r.s of the normal $(\overline{PR})$ are $\langle 3,-3,-2\rangle$.

So eqn of the plane is

$$3(x-2)+(-3)(y-1)+(-2)(z-1)=0$$

$$\Rightarrow 3x-6-3y+3-2z+2=0$$

$$\Rightarrow 3x-3y-2z-1=0.$$

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Q. Find the eqn of the plane parallel to the plane $2x - y + 3z + 1 = 0$ and at a distance 3 units away from it.

Sol^n

The required plane is parallel to the plane $2x - y + 3z + 1 = 0$

$ax + by + cz + k = 0$
$2x - y + 3z + k = 0$

So, eqn of the plane is $2x - y + 3z + k = 0$.

According to question distance between two planes is 3 unit.

$ax + by + cz + d = 0$.
$2x - y + 3z + 1 = 0$.

So

$$\text{distance}(d) = \left| \frac{d_1 - k}{\sqrt{(2)^2 + (-1)^2 + (3)^2}} \right| = 3$$

$$\Rightarrow \left| \frac{1 - k}{\sqrt{14}} \right| = 3$$

$$\Rightarrow \frac{1 - k}{\sqrt{14}} = \pm 3$$

$$\Rightarrow 1 - k = \pm 3\sqrt{14}$$

$$\Rightarrow 1 \pm 3\sqrt{14} = k$$

$$\Rightarrow k = 1 \pm 3\sqrt{14}$$

So eqn of required plane is

$$2x - y + 3z + 1 \pm 3\sqrt{14} = 0.$$

(13-b) (8-9) (10-11) (3-9 → 3,5,6, 7 imp), 8, 9, 10, 11, 12, 13, 14, 15 (1 imp) 16, 17, 18, 19, 20, 24, 27, 28

Equation of co-ordinate axes in Symmetric form:-

→ Since X-axis passes through (0,0,0) having d.c.s $\langle 1,0,0 \rangle$.

So eqn of X-axis in Symmetric form is $\Rightarrow \frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0}$

Similarly,

eqn of Y-axis in Symmetric form

$\Rightarrow \frac{x-0}{0} = \frac{y-0}{1} = \frac{z-0}{0}$,

& eqn of Z-axis in Symmetric form.

$\frac{x-0}{0} = \frac{y-0}{0} = \frac{z-0}{1}$.

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Application of derivative

★ ROLLE'S THEOREM:-

Let f be a function is

i) Continuous on $[a,b]$

ii) Differentiable on (a,b)

iii) $f(a) = f(b)$.

Then there exist atleast one point $c \in (a,b)$ such that $f'(c) = 0$.

Note:-

(i) → Every polynomial function is continuous and differentiable.

e.g:- $f(x) = 4x^3 - 5x^2 + 7x - 8$ is always continuous & differentiable.

eg Verify Rolle's Theorem for the function f defined by $f(x) = x^2 - 3x + 2$ on the interval $[1,2]$

Sol" Since $f(x) = x^2 - 3x + 2$ is a polynomial function, so, it is continuous on $[1,2]$ and differentiable on $(1,2)$

Now $f(1) = 1^2 - 3\cdot 1 + 2 = 0$

$f(2) = 2^2 - 3\cdot 2 + 2 = 0$

$\therefore f(1) = f(2)$

Since $f(x) = x^2 - 3x + 2$

$\Rightarrow f'(x) = 2x - 3$

$\Rightarrow f'(c) = 2c - 3$

Put $f'(c) = 0$

$\Rightarrow 2c - 3 = 0$

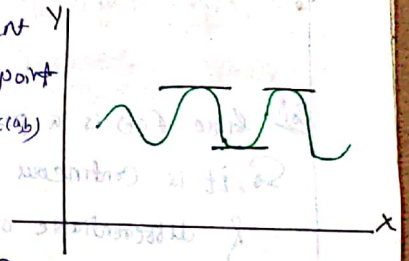
$\Rightarrow 2c = 3$

$\Rightarrow c = \frac{3}{2} = 1.5$

$\Rightarrow \frac{3}{2} \in (1,2)$.

∴ Hence Rolle's theorem is verified.

★ GEOMETRICAL INTERPRETATION OF ROLLE'S THEOREM:-



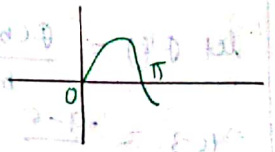
$f'(x)$ is the slope of the tangent to the curve $y = f(x)$ at the point $(c, f(c))$, where $a < c < b$ ($c \in (a,b)$)

$\Rightarrow f'(c) = 0$ implies that the tangent to the curve $y = f(x)$ at $(c, f(c))$ is parallel to the X-axis.

Thus if the function 'f' satisfy all the condition of Rolle's theorem on the interval $[a,b]$ then there exist atleast one point $(c, f(c))$ on the curve $y = f(x)$ where the tangent is parallel to X-axis.

eg $f(x) = \sin x$, $x \in [0, \pi]$ "verify Rolle's theorem."

$f(x) = \sin x$ is a continuous on $[0, \pi]$ & differentiable on $(0, \pi)$.



$f(0) = \sin 0 = 0$

$f(\pi) = \sin \pi = 0$

$\Rightarrow f(0) = f(\pi)$.

Since $f(x) = \sin x$

$f'(x) = \cos x$

$\Rightarrow f'(c) = \cos c$.

$\Rightarrow$ Put $f'(c) = 0 \Rightarrow \cos c = 0$

$\Rightarrow c = \frac{\pi}{2} \in (0, \pi)$.

Hence Rolle's Theorem is verified.

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☆ LAGRANGE'S (Mean value) THEOREM:-

→ If a function 'f' is
i) continuous on the close interval $[a,b]$.
ii) Differentiable on open interval (a,b)
then, there exist atleast one point $c \in (a,b)$.
Such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

eg) verify Lagrange's (mean value) theorem for the function $f(x) = 2x^2 - 3x + 7$ on the interval $[1,2]$.

Sol) Since $f(x)$ is a polynomial function
So, it is continuous on $[1,2]$.
& differentiable on $(1,2)$.

Now ~~$f'(x) = 4x-3$~~
~~$f'(c) = 4c-3$~~
$f(1) = 2\cdot 1^2 - 3\cdot 1 + 7 = 6$
$f(2) = 2\cdot 2^2 - 3\cdot 2 + 7 = 9$.

Now $f'(x) = 4x-3$
$f'(c) = 4c-3$

Let $f'(c) = \frac{f(b)-f(a)}{b-a}$

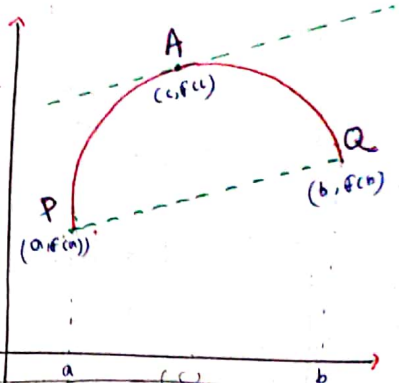
$\Rightarrow 4c-3 = \frac{-9-6}{2-1}$.

$\Rightarrow 4c-3 = \frac{3}{1} \Rightarrow 4c = 3+3$
$\Rightarrow c = \frac{6}{4} \Rightarrow c = \frac{3}{2} = 1.5 \in (1,2)$

∴ Hence Lagrange's mean value theorem is verified. $\rightarrow c \in (1,2)$

☆ GEOMETRICAL INTERPRETATION OF LAGRANGE'S MEAN VALUE THEOREM:-

We can find a point $A(c, f(c))$ on the curve where, $c \in (a,b)$.
Such that the tangent line at 'A' is parallel to the chord joining $(a, f(a))$ & $(b, f(b))$.



Imp Q.

☆ Note:-
→ Rolle's theorem is the particular case of Lagrange's mean value theorem.

Out of syllabus → * only for interested student.

☆ CAUCHY'S MEAN VALUE THEOREM:-

→ If f & g are 2 functions such that.
i) Both are continuous on $[a,b]$.
ii) Both are differentiable on (a,b).
iii) $g'(x) \neq 0$ for all $x \in (a,b)$.
Then there exist atleast one point $c \in (a,b)$ such that $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$

$\left\{ \because \frac{\frac{f(b)-f(a)}{b-a}}{\frac{g(b)-g(a)}{b-a}} \right\}$

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